Probability of Poker Hands

Drew Armstrong
armstron@math.umn.edu

November 1, 2006

In a standard deck of cards, there are 4 possible suits (clubs, diamonds, hearts, spades), and 13 possible values (2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace). Let A, J, Q, K represent Ace, Jack, Queen and King, respectively. Every card has a suit and value, and every combination is possible. Hence a standard deck contains $13 \cdot 4 = 52$ cards.

A "poker hand" consists of 5 unordered cards from a standard deck of 52. There are $\binom{52}{5} = 2,598,9604$ possible poker hands. Below, we calculate the probability of each of the standard kinds of poker hands.

Royal Flush. This hand consists of values $10, J, Q, K, A$, all of the same suit. Since the values are fixed, we only need to choose the suit, and there are $\binom{4}{1} = 4$ ways to do this.

Straight Flush. A straight flush consists of five cards with values in a row, all of the same suit. Ace may be considered as high or low, but not both. (For example, $A, 2, 3, 4, 5$ is a straight, but $Q, K, A, 2, 3$ is not a straight.) The lowest value in the straight may be $A, 2, 3, 4, 5, 6, 7, 8$ or 9. (Note that a straight flush beginning with 10 is a royal flush, and we don't want to count those.) So there are 9 choices for the card values, and then $\binom{4}{1} = 4$ choices for the suit, giving a total of $9 \cdot 4 = 36$.

Straight. A straight consists of five values in a row, *not* all of the same suit. The lowest value in the straight could be $A, 2, 3, 4, 5, 6, 7, 8, 9$ or 10, giving 10 choices for the card values. Then there are $\binom{4}{1}^5 = 4^5$ ways to choose the suits of the five cards, for a total of $10 \cdot 4^5 = 10,240$ choices. But this value also includes the straight flushes and royal flushes which we do not want to include. Subtracting the 40 straight and royal flushes, we get $10,240 - 40 = 10,200$.

Flush. A flush consists of five cards, all of the same suit. There are $\binom{4}{1} = 4$ ways to choose the suit, then given that there are 13 cards of that suit, there are $\binom{13}{5}$ ways to choose the hand, giving a total of $4 \cdot \binom{13}{5} = 5,148$ flushes. But note that this includes the straight and royal flushes, which we don't want to include. Subtracting 40, we get a grand total of $5,148 - 40 = 5,108$.

Four of a Kind. This hand consists of four cards of one value, and a fifth card of a different value. There are $\binom{13}{1} = 13$ ways to choose the value for the quadruple. Then, among the cards of this value, there are $\binom{4}{4} = 1$ ways to choose the quadruple. After this, there are $\binom{12}{1} = 12$ ways to choose a value for the single from the remaining values, and $\binom{4}{1} = 4$ ways to choose the single from the four cards of this value, for a grand total of

$$\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1} = 13 \cdot 1 \cdot 12 \cdot 4$$
$$= 624.$$

Full House. This hand consists of three cards of one value, and two cards of a different value. There are $\binom{13}{1}$ ways to choose a value for the triple, then $\binom{4}{3}$ ways to choose the triple from the four cards of this value. Then, there are $\binom{12}{1}$ ways to choose the value of the double from the remaining values, and $\binom{4}{2}$ ways to choose the double from the four cards of this value, for a grand total of

$$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 13 \cdot 4 \cdot 12 \cdot 6$$
$$= 3,744.$$

Three of a Kind. This hand consists of three cards of one value, and two more cards, each of different values. There are $\binom{13}{1}$ ways to choose the value for the triple, and $\binom{4}{3}$ ways to choose the triple from the four cards of this value. Then there are $\binom{12}{2}$ ways to choose two (unordered) values for the remaining singles, and $\binom{4}{1}\binom{4}{1}$ to choose the singles from their respective values, for a grand total of

$$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1} = 13 \cdot 4 \cdot 66 \cdot 4 \cdot 4$$
$$= 54,912.$$

Two Pairs. This hand consists of two pairs of different values, and a fifth card of another different value. There are $\binom{13}{2}$ ways to choose two (unordered) values for the two pairs, then $\binom{4}{2}\binom{4}{2}$ to choose the pairs from the cards of these values. Then there are $\binom{11}{1}$ ways to choose a remaining value for the single, and $\binom{4}{1}$ ways to choose the single from the four cards of this value, for a grand total of

$$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} = 78 \cdot 6 \cdot 6 \cdot 11 \cdot 4$$
$$= 123,552.$$

One Pair. This hand consists of a pair of one value, and three additional cards, each of different value. There are $\binom{13}{1}$ ways to choose a value for the pair, then $\binom{4}{2}$ ways to choose the pair from the four cards of this value. Then there are $\binom{12}{3}$ ways to choose three (unordered) values for the remaining three singles, and $\binom{4}{1}^3$ to choose suits for the singles, for a grand total of

$$\begin{aligned}\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3 &= 13 \cdot 6 \cdot 220 \cdot 4^3 \\ &= 1,098,240.\end{aligned}$$

Putting all of this together, we obtain the following ranking of poker hands:

Poker Hand	Number of Ways to Get This	Probability of This Hand
Royal Flush	4	0.000154%
Straight Flush	36	0.00139%
Four of a Kind	624	0.0240%
Full House	3,744	0.144%
Flush	5,108	0.197%
Straight	10,200	0.392%
Three of a Kind	54,912	2.11%
Two Pairs	123,552	4.75%
One Pair	1,098,240	42.3%
Nothing	1,302,540	50.1%

Wait, how did I compute the probability of getting “Nothing”?

How would you answer the question: “What is the probability of getting Three of a Kind or better?”