A \textit{q}-\textit{analogue} of Random-to-\textit{Random} Shuffling

\textbf{ILANI AXELROD-FREED}¹, JUDY HSIN-HUI CHIANG², VERONICA LANG³
MENTORS: BRAUNER, PATRICIA COMMINS⁴

M ASSACHUSETTS INSTITUTE OF TECHNOLOGY¹, UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN², SMITH COLLEGE³, UNIVERSITY OF MINNESOTA TWIN CITIES⁴

\textbf{MAIN RESULTS}

\textbf{R2R}^{(q)}_n \textit{Expression in End}(\mathbb{C}[G/B])

\begin{equation*}
\mathbf{R2R}^{(q)}_n = \sum \sum \ldots T_{i_j} \ldots T_{i_1} \bigotimes \mathbb{C}[G/B]
\end{equation*}

\textbf{Conjectural Explicit Formula and Representation Structure}

Let \( \lambda \) and \( \mu \) be a horizontal strip. \textit{We conjecture that all eigenvalues of} \( \mathbf{R2R}^{(q)}_n \) \textit{are of the form:}

\begin{equation*}
eig(\lambda/\mu)_q = \text{diag}(\lambda/\mu)_q + \sum_{j=1}^{\min(|\mu|,|\lambda|)} q^{j/\mu\setminus j/\lambda} [j/|\mu|]_q \in \mathbb{Z}_{\geq 0}[[q]],
\end{equation*}

\texttt{And for any eigenvalue} \( \epsilon \), \textit{the} \( \epsilon \text{-eigenspace has} \textit{q-Frobenius characteristic}

\begin{equation*}
d^\mu_{\epsilon q} \in \mathbb{C}[G/B]_{\epsilon q}
\end{equation*}

\textbf{PROOF STRATEGY: THE ALGEBRA OF WORDS}

Let \( M^{(n)} \) be the \textit{C}-vector space generated by words of length \( n \) in the alphabet \{1, 2, \ldots, n\} allowing repeated letters.

\textbf{Key idea:} In [4], Mathas defines the right action by \( H_n(q) \) on \( M^{(n)} \) in terms of rules for how each \( T_i \) acts on words. We can use this action to find recurrences and eventually solve for the eigenvalues of \( \mathbf{R2R}^{(q)}_n \).

\textbf{Example:} \( (121) \cdot T_3 = q(121) \) and \( (112) \cdot T_2 = (121) \).

\textbf{REFERENCES}


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