C.P.S. 4/3/2015

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Totally positive matrices with 1's on diagonal and upper triangular

$$n\times n \left\{ \begin{bmatrix} 1 & * & * & & * \\ & 1 & * & & * \\ & & \ddots & & \\ & 0 & & & * \\ & & & & 1 \end{bmatrix} \right. = e_{i_1}(t_1)\cdots e_{i_N}(t_N) \qquad t_i > 0$$

factor: if you ~~choose~~ fix a reduced word

$$w_0 = s_{i_1} s_{i_2} \cdots s_{i_N}, \quad N = \binom{n}{2}$$

where
$$e_i(t) = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & \boxed{\begin{matrix} 1 & t \\ & 1 \end{matrix}} & \\ & & & & 1 \end{bmatrix}$$
Chevalley generators

This factorization is unique once the reduced word is fixed.

Relations among $s_i = (i, i+1)$		Relations among $e_i(t)$
$s_i s_j = s_j s_i$		$e_i(a)e_j(b) = e_j(b)e_i(a)$
$s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}$	$\rightsquigarrow$	$e_i(a)e_{i+1}(b)e_i(c) = e_{i+1}(a')e_i(b')e_{i+1}(c')$
$s_i^2 = 1$		$e_i(a)e_i(b) = e_i(a+b)$

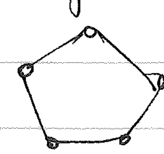
with $a' = \frac{bc}{a+c}$, $b' = a+c$, $c' = \frac{ab}{a+c}$

Now do the same for periodic n-T.P unitriangular matrices...

$$\left[\begin{array}{c|c} \begin{matrix} 1 & a & d \\ & b & \\ & & c \\ & & 1 \end{matrix} & \\ \hline & \begin{matrix} 1 & a & d \\ & b & \\ & & c \\ & & 1 \end{matrix} \end{array}\right]$$

$$e_1(a) = \left[\begin{array}{c|c} \begin{matrix} 1 & a & & 0 \\ & 1 & & \\ & & 1 & \\ & & \ddots & 1 \end{matrix} & 0 \\ \hline 0 & \begin{matrix} 1 & a & & 0 \\ & 1 & & \\ & & 1 & \\ & & \ddots & 1 \end{matrix} \end{array}\right]$$

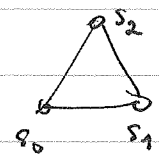
They satisfy the analogous relations for $\tilde{A}_{n-1}$



A qualitative difference: If $\sum_{j=1}^{\infty} t_j < \infty$, then $e_{i_1}(t_1)e_{i_2}(t_2)\cdots$ <u>converges</u>.

<u>Q</u>: When are 2 parametrizations equal??

<u>Braid limits</u>



$A_2^{(1)}$

$$e_2(a)e_1(b)\,e_2(c)e_0(d)\,e_1(e)\,e_2(f)\,e_0(g)\cdots$$

$$\underline{2\quad 1\quad 2}\quad 0\quad 1\quad 2\quad 0\quad\cdots$$

$$1\quad 2\quad \underline{1\quad 0\quad 1}\quad 2\quad 0\quad\cdots$$

$$1\quad 2\quad 0\quad 1\quad \underline{0\quad 2\quad 0}\quad\cdots$$

$$e_1(m)\,e_2(m)\,e_0(m)\quad 1\quad 2\quad 0\quad 2\quad\cdots$$

new parameter values!

Say $\underline{i} \to \underline{j}$ for two infinite reduced words if one can get from $\underline{i}$ to $\underline{j}$ by a braid limit. ~~Define~~ $\underline{i} \sim \underline{j}$ if $\underline{i} \to \underline{j}$ and $\underline{j} \to \underline{i}$.

Let $\Omega = \left\{ \prod_{j=1}^{\infty} e_{i_j}(t_j) : t_j \in \mathbb{R}_{>0} \right\}$

Given $X \in \Omega$,

consider <u>all</u> ways to write $X = \prod_{j=1}^{\infty} e_{i_j}(t_j) \rightsquigarrow \underline{i} = (i_1, i_2, \ldots)$

<u>CONJ</u>: $\exists$ a unique $\underline{i}$ in this set such that every other $\underline{i}$ in this set has $\underline{i} \to \underline{i}$