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<u>Super Catalan #'s</u>
Gessel

$$S(n,m) = \frac{(2n)!\,(2m)!}{n!\,m!\,(n+m)!}$$ $m=1 \rightsquigarrow 2\,(\text{Catalan } \#)$

· Known that $S(n,m) \in \mathbb{N}$

PROBLEM: What does it count?

Gessel/Xin $m = 0,1,2,3$ gave an interpretation
$m = n, n+1, n+2, n+3, n+4$ Chen/Wang 2012
In terms of lattice paths

<u>Super Narayana #</u>

$$A(m,n,k) = \frac{(2m)!\, n!\, (n-m)!}{m!\,(n-k)!\,(n-m-k)!\,(m+k)!\,k!} \in \mathbb{N}$$

$$\sum_k A(m,n,k) = S(m,n)$$

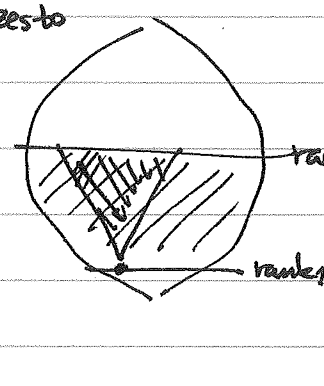
PROBLEM: What does it count?

<u>Generalizations</u>

B_k = Boolean algebra of rank k. Fix $x_1, x_2, \ldots, x_k$

$$\frac{\prod_{i=1}^{k} (2^{k-1} x_i)!}{\prod_{\emptyset \neq S \in B_k} \left(\sum_{i \in S} x_i\right)!} \in \mathbb{N} \quad \text{(Sundquist)}$$

generalizes to ? $\rightsquigarrow$



One might wonder:

Does $S_{2n} \times S_{2m}$ have a subgroup of order $n!\,m!\,(n+m)!$?

No, e.g. $n=2, m=5$

$S_4 \times S_{10}$ has no such subgroup of order $2!\,5!\,7!$

(Shareshian)