C.P.S. 9/26/2014 S. Sundaram

Genocchi numbers

$$-x \tanh \frac{x}{2} = \sum_{n \geq 0} (-1)^n \frac{G_{2n}}{(2n)!} x^{2n}$$

where $G_{2n} = \# \{ \sigma \in S_{2n-1} : \sigma(i) < \sigma(i+1) \iff \sigma(i) \text{ is even} \}$

CONJECTURE: There are positive integers $d_i(n)$ for $1 \leq i \leq n-1$ such that $G_{2n} = \sum_{i=1}^{n-1} \frac{d_i(n)}{2^i}$ and 2^i divides $d_i(n)$ $\forall i = 1, \ldots, n-1$

e.g.

$1(2)$ $G_4 = 1$

$1(2)$ $2(2^2)$ $G_6 = 3$

$1(2)$ $10(2^2)$ $6(2^3)$ $G_8 = 17$

$1(2)$ $36(2^2)$ $92(2^3)$ $26(2^4)$ $G_{10} = 155$

$1(2)$ $116(2^2)$ $840(2^3)$ $958(2^4)$ $158(2^5)$ $G_{12} = 2073$

There do exist $d_i(n) = 2(n-i)\, b_i(n) + (i+1)\, b_{i+1}(n)$ (defined recursively) where $b_i(n) =$ ~~dim of homology~~ # of times a certain permutation module occurs in top homology of partitions of $[2n]$ with even ~~# of blocks~~ Π^e_{2n}

$n=2$

Π^e_4 → 12/34 13/24 14/23 123/4 124/3 134/2 234/1

$$\tilde{H}(\Pi^e_{2n})\Big\downarrow_{S_{2n-1}} = \sum_{i=1}^{n-1} d_i(n)\, 1 \Big\uparrow_{S_2^i \times S_1^{n-i}}^{S_{2n-1}}$$