

# Reflection group counting and $q$ -counting

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# Outline

## 1 Lecture 1

- Things we count
- What is a finite reflection group?
- Taxonomy of reflection groups

## 2 Lecture 2

- Back to the Twelfefold Way
- Transitive actions and CSPs

## 3 Lecture 3

- Multinomials, flags, and parabolic subgroups
- Fake degrees

## 4 Lecture 4

- The Catalan and parking function family

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