Quiz 01.1

1. You should try decrypting the first 5-7 letters using different keys until you get recognizeable English, at which point you can decrypt the rest. The text was encrypted using a shift of 10; as I recall the plaintext is "The rain in Spain falls mainly on the garage."
2. 29. Note that $13 \cdot 29 = 377$, and $377\%47 = 1$.
3. I think it's hard to try to count the surjective functions directly, although it's possible. Instead, I reason like this: there are $8^2 = 64$ total functions from an 8-element set to a 2-element set. (You have two choices about where to send each of the 8 elements.) The only way a function to a 2-element set can *fail* to be surjective is if everything is mapped to the same element. There are two functions like that. Hence there are $64 - 2 = 62$ total surjective functions.
4. This is a harder question; you could try this to get some good practice at counting, but we won't have anything this hard on the test.

Quiz 02.1

1. Let's say we're mapping the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5, 6, 7\}$. An injective function sends each of the letters to a distinct number, so we need to choose four numbers, and *order matters*, since we first choose the target for a, then the target for b, etc. Hence there are $7 \cdot 6 \cdot 5 \cdot 4 = 840$ different injective functions.
2. $(a, b) = (5, 8)$
3. $\binom{13}{4} \cdot \binom{9}{5}$
4. This is a little more in-depth than you can expect on the text. Here's an overview of how you could do it. Let A be the event that you draw a blue ball, so A^C (the complement of A) is the event that you don't, i.e. the event that you draw a *red* ball. Then $P(A) = 4/7$ and $P(A^C) = (1 - 4/7) = 3/7$. You can use the formula on page 26 to find the probability of drawing exactly 6 blue balls. Then use the formula to figure ou the probability for exactly 7, 8, or 9 balls. Now add them up.

Quiz 03.1

1. 1/2. It's a fair coin, which means (by definition) that heads and tails are always equally likely on any given toss. (You could use the formula on page 26 to figure out just how likely or unlikely it is that you'll get 28 heads out of 71 flips of a fair coin.)
2. This is trickier than anything you'll have on the test, but it could be interesting for you to work on.

3. In cycle notation, P=(1 9 11 2 4)(3 14)(5 13 12 6 10 7 8). It turns out that P^4 is equal to (1 4 2 11 9)(5 10 13 7 12 8 6), although I just did this in my head and you should double check it!

4. In cycle notation, the given permutation is (1 7 2 11 9 10 6)(3 5 13 14 12)(4 8), which is a product of a 7-cycle, a 5-cycle, and a 2-cycle. The order of the permutation is the least common multiple of these lengths, which is 70.

Quiz 04.1

1. You can ignore this if you'd like; we didn't really talk about this sort of thing. We'll talk about it on Monday.

2. Here's a hint for this problem. A permutation has order twelve if that's the least common multiple of the lengths of its cycles. There are only a few possibilities:

   12-cycle: doesn't work, because we only have 8 objects, not 12.
   6-cycle and a 2-cycle: doesn't work because $lcm(6,2) = 6$, not 12.
   4-cycle and a 3-cycle: Aha! $lcm(4,3) = 12$.

   So now you're down to figuring out the following problem: how many different disjoint pairs of 4-cycles and 3-cycles can you create with 8 total objects?

3. With replacement, the probability of drawing a red ball at any time is $\frac{3}{7}$. Intuitively, we should expect about $\frac{3}{7} \cdot 13$ red balls in 13 draws.

4. This is an interesting problem, but work on the others first. Here's a hint: it has to be less than 13. (Why?)