Implicit in any homework problem is that you must explain why your answer is correct, even if the problem does not ask for a formal proof. Writing problems should have complete explanations of your work, written in complete sentences with correct grammar.

Due: Thursday, 10/27

Special Note: Make sure you heard or have seen the correction and clarification about accumulation points. This was done in lecture on Friday, 10/21. A page of notes will be posted on the course web page, but if you missed the class you might want to talk to somebody who was there to make sure you have everything that was covered.

Homework Assignment

Regular Problems:

(1) Identify the boundary points and accumulation points of each of the following sets. Also give the closure of each set. (By Definition 13.6 and Theorem 13.7, you can compute the closure of S as either $S \cup S'$ or $S \cup \operatorname{bd} S$.

(a) $A = (0, 1) \cup \{2\}$

(b) $B = \left\{ \frac{1}{n} \,|\, n \in \mathbb{N} \right\}$

(2) Determine whether each statement is true or false. Justify your answer using definitions and/or counterexamples. (You need not write any ε proofs to justify convergence for this problem.)

(a) If for any $\varepsilon > 0$ there exists an a_n such that $|a_n - a| < \varepsilon$, then $a_n \to a$.

(b) If $|s_n|$ converges to s, then s_n must converge to either s or $-s$.

(3) (This problem may not seem relevant to sequences at first glance, but leads to a handy technique we'll use repeatedly to show certain complicated sequences converge.) In this problem n is always a natural number.

(a) Find $K \in \mathbb{N}$ so that $n > K$ forces $n^2 + 10 < 2n^2$.

(b) Find $M \in \mathbb{N}$ so that $n > M$ forces $3n^3 - 16 > n^3$.

(c) Explain why, for some $N \in \mathbb{N}$, we have $\dfrac{n^2 + 10}{3n^3 - 16} < \dfrac{2n^2}{n^3} = \dfrac{2}{n}$. How can you find N based on K and M?

Writing Problem 1: Prove that $s_n = \dfrac{n^2 + 1}{n^2} \to 1$ using Defintion 16.2.

Writing Problem 2: Give an example of a sequence (a_n) of *rational* numbers which converges to $\sqrt{2}$, which is an irrational number. Prove that your sequence converges using Defintion 16.2. (Hint: in a previous week, we pointed out that if you truncate the decimal expansion of an irrational number like $\sqrt{2}$, the result is a rational number.)