Exam 2 covers Section 7 through Section 14, with the following caveats:

- Section 9 is not part of our course.
- The only thing you need to know from Section 14 is the following characterization from the Heine-Borel Theorem: *a subset S of $\mathbb{R}$ is compact if and only if it is closed and bounded.* Thus any questions about compact sets will actually be questions about sets which are closed and/or bounded.
- In Section 11, the ordered field $\mathbb{F}$ is not a central idea for our class and will not appear on this test. You are also not required to memorize the 15 axioms in this section.

The test itself will have a mixture of short answer questions, calculations and proofs.

The skills and writing problems are a good indication of what we consider important. Also see the list of review problems which was posted on the Moodle site to help you prepare for the review sessions led by TA's on Monday and Tuesday. Furthermore, you might recall that the solutions to the exams from the Spring 2009 semester are online. That course used a different textbook and therefore covered material in a slightly different order, but the following questions from the first exam are relevant when studying Sections 10 and 12 for our midterm.

Spring 2009 Exam 1. (Solutions at `http://www.math.umn.edu/~keynes/3283Exam1Solutions.pdf`.)

4. Use induction to prove $5^n + 6^n < 7^n$ for all $n > N$, for some N. (Hint: this fails for $n = 1$, for example, so you must determine the lowest value for which the statement holds, and use that as the basis for your induction.)

5(a). Let $A = \left\{ \frac{n-1}{n+1} \,\middle|\, n \in \mathbb{N} \right\}$.

   i. Show that A is bounded. (i.e. bounded above *and* below)
   ii. Find $L = \inf A$ and $M = \sup A$ (no proof needed for this part). Determine whether or not $L \in A$ and $M \in A$.
   iii. Prove the value $M = \sup A$ you found above is in fact the least upper bound of A.

5(b). Determine if the following sets are bounded above or below. In each case, if the set is bounded above, find the supremum. If the set is bounded below, find the infimum.

   i. $\left\{3 + \frac{1}{2}, -2 + \frac{1}{2}, 3 + \frac{1}{4}, -2 + \frac{1}{4}, 3 + \frac{1}{8}, -2 + \frac{1}{8}\right\}$.
   ii. $\left\{x \in \mathbb{R} \,|\, x > 0 \text{ and } x^2 - 4x + 3 > 0\right\}$
   iii. $\left\{x \in \mathbb{R} \,|\, x^3 - x < 0\right\}$
   iv. $\{1 - .3, 2 - .33, 3 - .333, 4 - .3333, 5 - .33333\} \cup \left\{ \frac{1}{\sqrt{n}} \,\middle|\, n \in \mathbb{N} \right\}$

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