

The following is a non-comprehensive list of solutions to the skills problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

1.4 Your answers may vary according to whether you know and use the antonyms to some of the terms being negated. For example, “not bounded” is also referred to as “unbounded.”

- (a) The relation R is not transitive. (Or, the relation R is intransitive.)
- (b) The set of rational numbers is not bounded. (Or, the set of rational number is unbounded.)
- (c) The function f is either not injective or not surjective.
- (d) $x \geq 5$ and $x \leq 7$.
- (e) x is in A and $f(x)$ is in B .
- (f) f is continuous, and $f(S)$ is either not closed or not bounded.
(Or, f is continuous and $f(S)$ is either not closed or unbounded.)
- (g) K is closed and bounded and K is not compact.

- 1.10 (c) False, because both statements (6 is prime, 8 is odd) are false.
- (e) True, because the antecedent ($2 + 2 = 5$) is false, so the whole implication is true.
- (g) True for the same reason as (e); the antecedent (5 is odd and 6 is prime) is false because 6 is not prime.
- (i) False, because the antecedent is true ($2 + 5 = 7$ and $2 \cdot 5 = 10$) but the consequent is false.
- (j) True. The double negatives make this difficult to read; in symbols we have

$$\sim [(4 \text{ is even}) \wedge \sim (7 \text{ is prime})]$$

Using De Morgan's Law, $\sim [p \wedge q] \Leftrightarrow (\sim p) \vee (\sim q)$, this is equivalent to

$$(4 \text{ is odd}) \vee (7 \text{ is prime})$$

This disjunction is true, because the second statement is true.

- 1.12 (a) $\sim p \wedge q$
- (b) $(\sim p) \wedge (\sim q)$. This is also $\sim (p \vee q)$
- (c) If q , then $\sim p$. In symbols, $q \Rightarrow \sim p$.
- (d) The statement “ p is necessary for q ” is equivalent to $q \Rightarrow p$. In general, if the implication $s \Rightarrow n$ is true, then s is a *sufficient* condition for n (because if s is true, n must be true as well), whereas n is a *necessary* condition for s , since s can't be true without n being true as well.
- (e) The statement “ q only if p ” is equivalent to $q \Rightarrow p$. One way to see that is to observe that the only way this statement could be false is if Buford passes the class but does not get a C on the exam—in other words, if q is true and p is false. If you look at the truth table for an implication, this exactly describes the truth values of $q \Rightarrow p$.