

Due: 11 October 2012 at the beginning of class. You will not have a writing quiz that day.

Your solution should be written out carefully and will be graded according to the rubric on the course page. Because you are not under the time pressure of a writing quiz, your work should be especially well organized; in particular you should write out at least one draft of your solution before you write your final draft to hand in. Although you can rewrite your solution after it is graded, both drafts will count towards your grade.

You can (and should) work with others on your solution, but your final solution must be your own, written in your own words. If your solution is taken from an online or printed resource you will receive a zero on both your initial draft and rewrite.

The Schröder-Bernstein Theorem, mentioned in Exercise 8.13, says if there exists an injection $f : A \rightarrow B$ and an injection $g : B \rightarrow A$, then there exists a *bijection* $h : A \rightarrow B$. Equivalently, using our definition of cardinality,

$$\text{If } |A| \leq |B| \text{ and } |B| \leq |A|, \text{ then } |A| = |B|.$$

You do not have to prove this theorem, but you can make use of it to solve the following problems.

- (1) Prove Theorem 8.15(a,b,c,d). [You may leave out part (e).]
- (2) Use Theorem 8.15 and the Schröder-Bernstein Theorem to provide a simpler proof than the one given in class that $[0, 1] \sim [0, 1)$.
- (3) Use Theorem 8.15 and the Schröder-Bernstein Theorem to prove $(0, 1) \sim \mathbb{R}$.