Due: 15 November 2012 at the beginning of class. You will not have a writing quiz that day.

Your solution should be written out carefully and will be graded according to the rubric on the course page. Because you are not under the time pressure of a writing quiz, your work should be especially well organized; in particular you should write out at least one draft of your solution before you write your final draft to hand in. Although you can rewrite your solution after it is graded, both drafts will count towards your grade.

You can (and should) work with others on your solution, but your final solution must be your own, written in your own words. If your solution is taken from an online or printed resource you will receive a zero on both your initial draft and rewrite.

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(1) Consider the sequence $(s_n) = \left(\dfrac{2n^2+2n}{3n^2-4}\right)$. Use the limit definition of convergence (16.2) to prove $s_n \to \dfrac{2}{3}$.

Hints: recall from lecture that your goal is to find a simpler sequence $a_n = \dfrac{p(n)}{q(n)}$ which converges to 0 and which (eventually) bounds $\left|s_n - \dfrac{2}{3}\right|$ from above:

$$\exists\, m \in \mathbb{N} \text{ such that } n > m \Rightarrow \left|s_n - \frac{2}{3}\right| \leq a_n$$

This can be accomplished by choosing $p(n)$ and $q(n)$ such that

$$p(n) > \text{numerator of } s_n - 2/3$$
$$q(n) < \text{denominator of } s_n - 2/3$$

(Note the different directions in the inequalities!) You will make your life much simpler if your $p(n)$ and $q(n)$ both have the form $c \cdot n^k$, i.e. a power of n multiplied by some coefficient. Then use the fact that $a_n \to 0$ to finish your proof. Overall this is very similar to Example 16.6, which we covered in detail in class. Example 16.9 has similar steps as well.

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