Math 3283W Fall 2012 - Writing Quiz #2

Thursday 9/20/12 - 20 minutes Name: Solutions

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (7 Points) Prove or give a counterexample to the following statements.

(a) If $x + y$ is irrational, then x is irrational or y is irrational.

We prove the contrapositive: "if x and y are rational, then $x+y$ is rational". Suppose x and y are rational. Then $x = \frac{m}{n}$, $y = \frac{p}{q}$ where m, n, p, q are integers and $n \neq 0$, $q \neq 0$. We add:

$$x + y = \frac{m}{n} + \frac{p}{q} = \frac{mq + pn}{nq}.$$

The sum and product of two integers are again integers, so $mq + pn, nq \in \mathbb{Z}$ (and $nq \neq 0$); thus $x+y$ is rational, establishing the contrapositive and hence the claim itself.

(b) If xy is rational, then x is rational or y is rational.

The statement is <u>false</u>, since if $x = y = \sqrt{2}$, $xy = \sqrt{2} \cdot \sqrt{2} = 2$ is rational, but neither x nor y is rational.

(2) (7 Points) Prove or give a counterexample: The sum of any four consecutive integers is never divisible by four.

The claim is <u>true</u>. Any four consecutive integers may be expressed as n, $n+1$, $n+2$, and $n+3$ for some integer n. Their sum is then

$$n + (n+1) + (n+2) + (n+3) = 4n + 6.$$

But $4n + 6$ is never divisible by four, for any $n \in \mathbb{Z}$: if it were, $\frac{4n+6}{4} = n + \frac{3}{2}$ would be an integer, an absurdity since n is an integer and $\frac{3}{2}$ is not. We conclude <u>no</u> four consecutive integers have a sum divisible by four.