This exam covers Section 1.1 through Section 2.2; there is no need to take material from Section 2.3 to write a full length exam. Midterms in Math 3283W generally have about 4-6 pages of problems, which may seem long for a 50-minute exam. The point is that the exam is checking how well you have mastered the material and, if you've learned it well, you can not only do the problems but do them fairly quickly. In other words, if you're really comfortable looking at a sentence, rewriting it in mathematical and logical notation, writing its negation (or converse, or contrapositive, or whatever...), then a problem along those lines should take no more than a few minutes. If you can do those sorts of problems, but it takes 10 minutes of thinking things through, looking up a symbol in the book, checking the definition of converse, etc., then you should practice these problems until you can do them faster.

Similarly, if you're comfortable doing direct, contrapositive or contradiction proofs using even/odd or rational/irrational numbers, you'll be able to do any similar problem on the test in a few minutes, even though it's a "proof problem." If it takes you 5-10 minutes to sort out what the contrapositive statement is, you'll risk using up too much time on a problem of that sort.

## How to Study

Exams in Math 3283W are different than exams in the Calculus sequence, which are mostly computational. We haven't done many computations in this class, so the test will focus on whether you understand and can use definitions (equivalence relation, truth table, etc.) and if you've learned the various techniques of proof discussed in class. To study for this test you should learn the definitions and theorems, but there is no substitute for *doing problems*. You can understand how to prove two sets are equal to each other, but if you haven't done many of those proofs on your own, you'll be hard pressed to do so in a timely manner on the exam.

You can look at your lecture notes, homework assignments and writing quiz practice problems to get a good feel for what topics I feel are important and are likely to appear on the test. Redo any homework problems that you struggled with. There are 11 office hours available to you before the exam takes place; use them to ask about those problems, or any concepts that haven't made sense. At first you should use your textbook, notes and other resources if you're stuck, but your eventual goal is to be able to solve these problems without using any help.

Also remember that the book has lots of similar exercises which weren't assigned, providing a good source of additional problems for your studying. In each section, the true/false questions at the beginning of the section are good way to refresh your memory about the section. If you'd like to see actual past exam problems, here are a few from the Spring 2009 semester; answers can be found at http://www.math.umn.edu/~keynes/3283Exam1Solutions.pdf. Keep in mind that the Spring 2009 course was run out of a different textbook, so the notation, material and order in which things were covered were sometimes a bit different. Problems 4-6 on the Spring 2009 exam cover material that we haven't seen yet.

1. Prove that the following pairs of statements are equivalent or give a counterexample to show they are not.
   (a) $(P \wedge Q) \vee R$, compared to $P \wedge (Q \vee R)$.
   (b) $(P \wedge Q) \Rightarrow R$, compared to $(P \Rightarrow R) \vee (Q \Rightarrow R)$.
2. Write the following statements using mathematical and logical symbols, and using the mathematical expression $P(x, y) : x < y$ where possible. Your statements should not use the symbols $<$ or $>$.
   (a) Given any two real numbers $a$ and $b$ with $a < b$, their average $\dfrac{a+b}{2}$ is greater than $a$ but less than $b$.
   (b) Given any positive number less than 1, its square is less than itself.
   (c) The negation of (b).
3. Consider $p$: If $A$ and $B$ are subsets of $\mathbb{R}$ such that $A \cap B$ is infinite, then both $A$ and $B$ are infinite.
   (a) Write the contrapositive of $p$.
   (b) Write the converse of $p$.
   (c) Is the converse of $p$ true or false? Justify your answer with either a proof or a counterexample.