The writing exercises below will form the basis for the Writing Quiz on 10 October 2013.

(1) Let $f : A \to B$ and $g : B \to C$.
   (a) Prove: if f and g are injective, then $g \circ f$ is injective.
   (b) Prove: if f and g are surjective, then $g \circ f$ is surjective.

(2) Let $f : A \to B$ and $C_1, C_2 \subset A$.
   (a) Prove $f(C_1 \cup C_2) = f(C_1) \cup f(C_2)$.
   (b) Prove $f(C_1 \cap C_2) \subseteq f(C_1) \cap f(C_2)$.
   (c) If f is bijective, prove $f(C_1 \cap C_2) = f(C_1) \cap f(C_2)$

   Hint: for the previous problem it might help you understand what's going on if you choose a function like $f(x) = x^2$ or $f(x) = \cos x$ and compute the sets for various choices of C_1 and C_2. As always, these examples won't constitute a solution; you need to write a general proof.

(3) Let $f : A \to B$ and $D \subseteq B$. Prove $f^{-1}(B \setminus D) = A \setminus f^{-1}(D)$.

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