How often have you complained that a textbook is difficult to read or doesn't explain things well enough? This writing project is your chance to do better! Your assignment is to write a short section on continuous functions for a pretend textbook. After writing a few sentences to introduce the topic, you'll state a definition (given to you below) of when a function f is continuous at $x = c$. You'll prove a few theorems and provide examples as appropriate to help the reader understand the material. This project touches on both of the goals we mentioned during the first week of Math 3283W: to learn the fundamentals of mathematical proof and writing, and to begin to put Calculus on a more rigorous footing; the definition of continuous function is one of the first important definitions involving limits in Calculus I.

Below you will find the schedule for the project and a description of the requirements. Until now the writing component of this course has been limited to (relatively) short proofs, so this packet also includes a number of tips for writing a longer mathematical document. The peer review form and grading rubric for your drafts will be posted on the course website for your reference.

Project Schedule

Here are the relevant dates and deadlines for your project:

Tuesday 11/12/13, Thursday 11/14/13 and Tuesday 11/19/13

In addition to talking about skills problems and other topics, you'll spend time during these classes working through the mathematics for your project. Some of the required proofs will be covered during lecture as well.

Thursday 11/21/13

Peer review day. Bring **TWO** copies of your first draft, one for your TA's reference and one for another student in the class to read through. Your TA will guide you through the peer review process. You will receive 8 points for your own draft (more for completeness than mathematical correctness or writing excellence at this point) and 8 points for your careful, considered review of another student's work. Overall this process will be worth **16 writing points**, or the equivalent of one regular-length writing quiz.

Tuesday 11/26/13

Hand in your second draft in class; this is the first time your work will be directly evaluated by your TA. This draft should be typed; see the tips below about possible tools you could use. It will be handed back to you the following week with comments and a score from your TA. This draft will be worth **16 writing points**, or the equivalent of one regular-length writing quiz.

Tuesday 12/10/13

Hand in your final (typed!) draft in class. This final draft will be worth **16 writing points**, or the equivalent of one regular-length writing quiz.

Please follow any directions from your TA when handing in drafts. For example, they may request that you hand in your second draft along with the final draft, since it can help them quickly determine how well you have addressed the comments they made about your earlier work.

Project Requirements

As described above, you will write a short section on continuous functions for a textbook. At the end of this semester we may describe the so-called "ε-δ definition of continuity," but we've done enough with sequences to work with the definition below.[1]

> **Definition.** Let $D \subseteq \mathbb{R}$. A function $f : D \to \mathbb{R}$ is continuous at $c \in D$ if, for every sequence x_n in D which converges to c, the sequence $f(x_n)$ converges to $f(c)$. In symbols, assuming $x_n \in D$ for all n,
> $$x_n \to c \quad \Rightarrow \quad f(x_n) \to f(c).$$
> If f is continuous at every point in its domain, we simply say f *is continuous.*

Your "textbook section" must contain the following:

(1) A short introduction.

(2) The definition of continuity given above.

(3) A definition of what it means for f to be discontinuous at c. (Hint: negate the definition!)

(4) Examples of continuity and discontinuity as appropriate to help the reader understand the concepts.

(5) Proofs of the following important theorems. (Hint: the proof of the first theorem essentially amounts to citing Theorem 4.2.1 in your book, which you may do.)

> **Theorem 1** Let f and g be functions from $D \subseteq \mathbb{R}$ to $\mathbb{R}$ and let $c \in D$. Suppose that f and g are continuous at c. Then:
> (a) $f + g$ and fg are continuous at c
> (b) $k \cdot f$ is continuous for any $k \in \mathbb{R}$.
> (c) f/g is continuous at c if $g(c) \neq 0$.
>
> **Theorem 2** Let f and g be continuous real-valued functions such that the range of f is contained in the domain of g. Then the composition $g \circ f$ is continuous.

(6) A final example or two, illustrating those theorems as appropriate.

The parts of your section need not be in this exact order, but you will be evaluated on your organization: have you arranged your theorems and examples in a logical and easy-to-follow progression which helps your reader understand the topics?

Similarly, you do not need to copy the above definition and theorems word-for-word and symbol-for-symbol. Feel free to rephrase and improve them, but your statements must be mathematically equivalent to the definition and theorems above.

As with all assignments in this class, you are encouraged to work with other students, but you must write your proofs and examples in your own words. When you hand in drafts you will be asked to identify any collaborators (and which sections they are in) along with any outside sources you used as a reference. We are required to report all instances of plagiarism; please do not put us in that position, which would not be a pleasant experience for you or for us.

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[1] This is actually the definition for *sequentially continuous*, but it turns out to be equivalent to *continuous* when working with functions from (subsets of) $\mathbb{R}$ to $\mathbb{R}$. You'd have to take certain 5000 or 8000 level courses to see situations where *sequentially continuous* and *continuous* turn out to be different.

Writing Tips

The grading rubric for your project will be posted online, but please be aware of the following. On a timed writing quiz or exam we can be somewhat lenient about grammar and related issues. On a larger scale writing project, where the work is done outside of class in an untimed environment, you are expected to write grammatically correct English and to follow the conventions of mathematical writing which have been discussed this semester. **For full credit, your writing should**: consistent of full sentences; use correct spelling, punctuation, capitalization and grammar; have sentences which begin with words, not mathematical expressions; use symbols appropriately; indent and arrange mathematical blocks in an appropriate way; and be legible.

The following advice for writing a mathematical document is adapted from guidelines written by Prof. David Clark (`dcclark@umn.edu`) and other instructors in the University of Minnesota Talented Youth Math Program (UMTYMP).

Writing math for real people. Despite the saying that "mathematics is the universal language", no one in the world actually speaks *math*. Mathematical ideas must be written out in a natural language and read by real people. We've all had the frustration of trying to understand a poorly written textbook. The burden of communication lies with *you*, the writer, not with the reader. Here are some basic principles which help with writing math for real people.

(1) Be concise. Write a solution in the most direct way. This can be surprising: The best way to write up a solution may be completely different than the way that you initially found it. You may have to completely restructure and rewrite your work. **DO IT!** Readers should be able to easily see that your work is correct, and *not* have to follow the twists and turns of your own discovery process.

(2) Give your objects meaningful names. Be aware of naming conventions. There is nothing technically wrong with saying "Let $x : \mathbb{R} \to \mathbb{R}$" but everyone expects x to be a real number, not the name of a function; we generally names functions f, g or h. If you use non-standard names for objects it will be harder for readers to make sense of the new material.

(3) Italicize all mathematical variables and functions. This is a visual hint that you're looking at math, not text. **This seemingly minor typesetting issue can make a huge difference in the quality of your document.** You may never have realized this, but the mathematics in your textbooks is italicized and often written in a different font with different spacing. Notice how 2x+3y+4z=6 looks odd; it should be typeset as $2x + 3y + 4z = 6$. See the difference?

    If you write in LaTeX or use the equation editors in Microsoft Word or Google Docs, the italicizing will be done automatically for you. But it can be a shock to realize that you need to use the equation editor for every single mathematical expression, even if you just mention the variable x in passing.

(4) Give important equations their own line (and number them if you need to refer to them later). Less important equations, or intermediate calculations, can be typed inline.

(5) When writing a long chain of equalities, write each equality on its own line, and line up the "="s:

$$\begin{aligned} \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} &= \lim_{r\to 0} \frac{r\cos(\theta) r\sin(\theta)}{r} \\ &= \lim_{r\to 0} r\cos(\theta)\sin(\theta) \\ &= 0 \end{aligned}$$

    This makes it easier to follow each step. A very brief word of explanation can go on the right of an equation if necessary. For more detailed explanation, interrupt the equalities and give the explanation in a full sentence on its own line.

(6) Include diagrams and figures when necessary. They become "necessary" when you find yourself spending more lines describing a situation than the figure would take up on its own! It is often said that "a picture is worth a thousand words," and in math this is especially true.

(7) Use complete sentences with capital letters, periods, proper punctuation, nouns, and verbs. Mathematical equations included in sentences are part of the sentence as well. For instance "As a result, $x = 2$." is a complete sentence, beginning with a capital, ending with a period.

(8) Don't confuse words and symbols: Never use a mathematical symbol when you mean the word for which it stands. For example, say "The area is 5" or "Then $A = 5$", but *never* say "So the area = 5". The same goes for arrows: say "The area is positive, which implies that $x = 5$", not "$A > 0 \to x = 5$". See the comment at the top about nobody actually *speaking* math!

(9) Show appropriate work. Provide enough structure to your writing to allow your reader to reconstruct your work on their own without undue burden.

(10) On the other hand: Know your audience. When writing for Math 3283W students, you don't need to explain arithmetic in great detail (if you do, you'll aggravate them and make them want to stop reading!).

(11) Clearly state your hypothesis and conclusions, both in proofs and in examples. Ignoring these is like writing an essay without an introduction or conclusion and makes examples very hard to follow.

(12) Similarly, define all functions and variables. This may be done with a sentence ("Let b be the length of the base of triangle T."), in an equation ("Let $A = \pi r^2$, where r is the radius of circle C"), or visually in a figure (in which case, state in a sentence that "Let n be defined as in Figure 1.1"). A very common mistake is to assume that certain commonly used names for variables and functions have inherent, universal meaning. Do *not* assume that s refers to the limit of whatever sequence you are working with, or that your readers will automatically know this. Instead, define it!

(13) Revise, revise, revise! Never hand in the first copy of *anything*, even this first draft! Step back for an hour, reread your solution, rethink it, and rewrite it in a better way.

Technical Tips. Your initial draft can be handwritten, but must be legible. The remaining drafts must be typed, and you may wish to get a head start on this process by typing your first draft as well. You have at least four options for how to type a mathematical document with equations, variables and other mathematical expressions:

- Use any word processor to type your text, leaving blank spaces for all your mathematical expressions and writing them in VERY NEATLY by hand. This tedious process was the norm for decades, but is not recommended. It does not scale well to multiple drafts. Every time you change the text on a page, you have to rewrite the mathematics by hand.

- Microsoft Word has an Equation Editor which suffices for most mathematical writing. Recall item 3 in the list above about using the editor for all mathematical expressions.

- Google Docs also has a serviceable equation editor; a short tutorial is available at:

  https://support.google.com/drive/answer/160749?hl=en

  This option has the additional advantage that you can share the document with any UMN user—your TA, for example. **This is our recommended approach for writing your project if you are not already well versed with one of the other methods.**

- Mathematicians generally use LaTeX to prepare their documents; see http://www.latex-project.org/ or http://www.tug.org/texlive/ for more information. Installing LaTeX on your own computer requires a LARGE download, but you can use an online LaTeX system such as http://www.writelatex.com. If you are planning on a career in mathematics, science or engineering it is probably worth learning LaTeX, but there is a steep learning curve, so we would **not** recommend you start learning it from scratch just for this project.