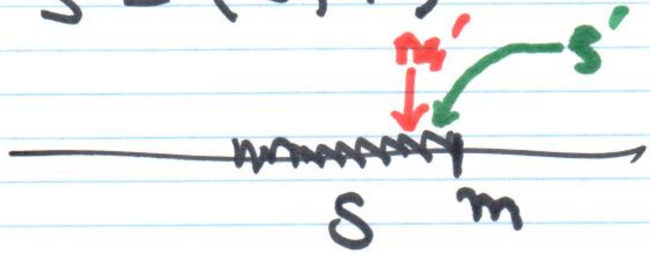
MATH 3283W

10/23/13

$S = (0, 1)$



Contrap. of 2 :

If $m' \in \mathbb{R}$ s.t. $s \leq m'$, $\forall s \in S$
then $m \leq m'$.

Suppose $m_1 = \sup S$
$m_2 = \sup S$

$\longrightarrow m_1 \leq m_2$ ($m_1 = \sup S$, m_2 u.b.)

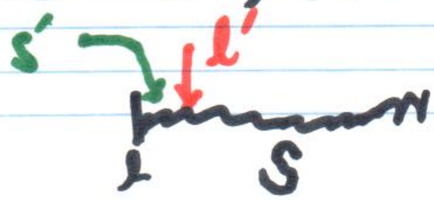
$\longrightarrow m_2 \leq m_1$ ($m_2 = \sup S$, m_1 u.b.)

$\longrightarrow m_1 = m_2$.

$l = \inf S$ if

1) $\forall s \in S$, $l \leq s$.

2) If $l' > l$, then $\exists s' \in S \ni s' < l'$



$$S = \left\{ 0, \frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \cdots \frac{998}{1000}, \cdots \right\}$$

$$\longrightarrow 1$$

For $n \in \mathbb{N}$,

$n-1 \geq 0$	$n-1 < n+1$
$n+1 > 0$	$\frac{n-1}{n+1} < 1$
$\frac{n-1}{n+1} \geq 0$	
0 is l.b.	1 is u.b.

To show $1 = \sup S$.

Show $\forall \varepsilon > 0$, $1 - \varepsilon$ is not an upper bound.

Show $\exists n \ni \frac{n-1}{n+1} > 1 - \varepsilon$.

<u>Scratch</u>

$$\varepsilon > 1 - \frac{n-1}{n+1}$$

$$= \frac{n+1}{n+1} - \frac{n-1}{n+1}$$

$$= \frac{2}{n+1}.$$

$$\hookrightarrow n+1 > \frac{2}{\varepsilon} \longrightarrow n > \frac{2}{\varepsilon} - 1$$

Choose n so large that $n > \frac{2}{\varepsilon} - 1$.

<u>Proof</u> To show $1 = \sup S$,

show $\forall \varepsilon > 0$, $1-\varepsilon$ not u.b.

Given $\varepsilon > 0$.

Choose $n > \frac{2}{\varepsilon} - 1$. ⟵ WHY? Arch. prop.

$$n+1 > \frac{2}{\varepsilon}$$

$$\varepsilon > \frac{2}{n+1} = \frac{(n+1)-(n-1)}{n+1}$$

$$= 1 - \frac{n-1}{n+1}$$

$$\frac{n-1}{n+1} > 1 - \varepsilon.$$

↳ $\in S$. $1-\varepsilon$ NOT u.b.

$0 \in S$ $(n=1)$.

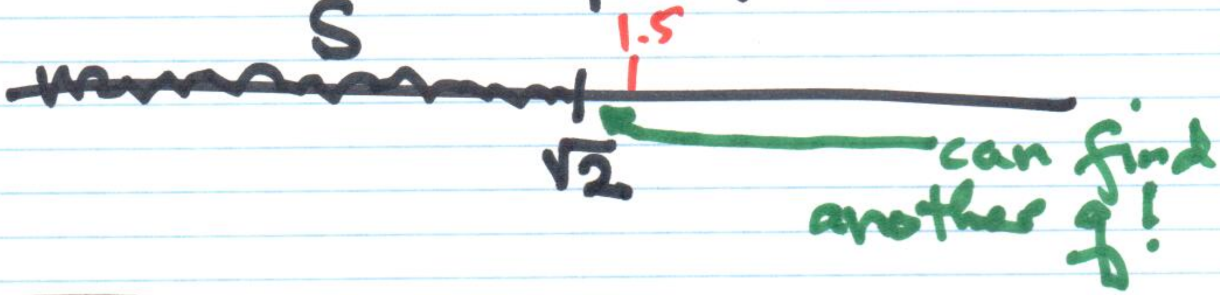
↳ $0 = \inf S$.

If $S = \emptyset$, every $x \in \mathbb{R}$ is an upper bound:

$\forall s \in \emptyset$, $s \le x$. TRUE.

$S = (-\infty, \sqrt{2}) \cap \mathbb{Q}$

No upper bd $q \in \mathbb{Q}$
is a sup. for S.



Archimedean prop.

<u>Pf</u> Suppose, for sake of contradiction,
$\mathbb{N}$ bdd above in $\mathbb{R}$.

Completeness axiom $\rightarrow$
$m = \sup \mathbb{N}$.

$m-1$ is not u.b.

$\exists\, n \in \mathbb{N} \ni m-1 < n$
$\rightarrow m < n+1$. ✱

$\mathbb{N}$ unbdd above.

3. $x =$ size of steps
$y =$ finish line
$n =$ # steps

4. Capture the flag.