§1.2 Quantifiers

"Quantifers"..... quantify things. (!!)

In math/logic we mostly care if a statement is true

- always $\qquad x^2 \geq 0$
- at least once ("sometimes") $\qquad x^2 > 0$
- never. $\qquad x^2 < 0$

not really - efficient!

Since we're lazy we use symbols....

Existential Quantifiers

$\exists$: there exists (at least one), there is a

$\exists!$: there exists a unique (exactly one)

$\nexists$: there does not exist (slang....)

Universal

$\forall$: for all, for every.

Other notation

$\ni$: such that (some people use :)

$p(x)$: stmt p whose truth value depends on x.

Ex $\quad p(x): x^2 - 1 = 0$ $\qquad$ $p(0)$ is false

$p(1)$ is true.

Ex Write these stmts in symbols

For some x, $x^2-1=0$

$\exists x \ni x^2-1=0.$

For every real number $x>0$, there is a real number y such that $y^2=x$.

$\forall x>0\ \exists y \ni y^2=x.$ OR $\forall x>0\ \exists y\in\mathbb{R} \ni y^2=x.$ OR ...

Every real # has a cube root

$\forall x \in\mathbb{R}\ \exists y \ni y^3=x.$ $y=\sqrt[3]{x}$

For every # there is a larger #.

$\forall x\ \exists y \ni y>x.$

There is a largest real #: $\exists y \ni \forall x,\ y>x.$

two more

If $x>1$, then $x^2>1$.

$\forall x>1,\ x^2>1.$

There is no square root of -2 in $\mathbb{R}$.

$\not\exists x \ni x^2=-2.$ (slang....)

$\forall x\in\mathbb{R},\ x^2\neq -2.$

⚠ Negation of stmts with quantifiers is tricky.

In words: negation of "every day is sunny" isn't "every day is cloudy."

it's "at least one day is cloudy"

or "there exists a cloudy day."

(Here $\sim$sunny = cloudy)

Symbolically

- Negation of $\forall x,\ p(x)$ is $\exists x \ni \sim p(x)$.

i.e. $$\boxed{\sim[\forall x,\ p(x)] \Longleftrightarrow \exists x \ni \sim p(x)}$$

- Also, $$\boxed{\sim[\exists x \ni p(x)] \Longleftrightarrow \forall x,\ \sim p(x)}$$

Ex Negate:

(a) $\forall x,\ g(x) > 0$ $\quad \exists x \ni g(x) \le 0 \quad (\exists x \ni \sim g(x) > 0)$

(b) $\exists x \ni f'(x) = 0$ $\quad \forall x,\ f'(x) \neq 0.$

(c) $\forall \varepsilon > 0\ \exists \delta > 0 \ni$ ∀x with $0 < |x-a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon.$

$\exists \varepsilon > 0 \ni \forall \delta > 0\ \exists x \ni 0 < |x-a| < \delta$ and $|f(x) - f(a)| \ge \varepsilon$

Inchworm on a Rope Solution

Keep track of %age along rope.

1s: 
1cm
1m = 100cm $\frac{1}{100}$


2cm
2m $\frac{1}{100}$

2s 
3cm
2m = 200cm $\frac{1}{100} + \frac{1}{200}$


4.5 cm
3m

3s 
5.5 cm
3m = 300 cm $\frac{1}{100} + \frac{1}{200} + \frac{1}{300}$

After n seconds, $\left(\frac{1}{100} + \frac{1}{200} + \cdots + \frac{1}{n \cdot 100}\right) = \frac{1}{100}\underbrace{\left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right)}_{\text{eventually reaches 100}}$

$\sim 10^{42}$ sec. ←