§2.1 Basic Set Theory

Set thy seems tedious at first, but it's essential.
In higher level math courses, the language of set theory "replaces" arithmetic.

★ Read this section carefully, even ★
★ (or especially?) if you know some ★
★ set thy. ★

Def A set is an unorderd collection of objects, called elements. Write $x \in A$ to denote x is an element of A. If A has a finite # of elts, $|A| = \#$ of elts in A
$=$ cardinality of A.

Ways to Define Sets

list elements

$A = \{1, 2, 3, 4, 5\}$

$B = \{\circ, \triangle, \square\}$

defining property

$C = \{x \mid x > 0\} = \{x : x > 0\}$ ($= \{x > 0\}$ "sloppy")

Note A "universal set" is often implied or assumed.

A: integers? B: shapes? C: real #?

Std Names

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ = natural #'s

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ = integers.

$\mathbb{Q}$ = rat'l #'s

$\mathbb{R}$ = real #'s

$\mathbb{C}$ = complex #'s.

$\mathbb{F}_{p^n}$ = finite field w/ p^n elements.

$\mathcal{C}^1$ = fns w/ cont. deriv.

$\emptyset = \{\}$

$(1, 3] = \{x \mid 1 < x \leq 3\}$

Def A is a subset of B, $A \subseteq B$, if $\underline{x \in A} \Rightarrow x \in B$.

└ always F if $A = \emptyset$

Ex $B = \{1,2,3,4\}$

non-proper $\{1,3,4,2\} \subseteq B$

proper $\{1,3\} \subseteq B$

$\{1,2,5,6\}$ no! $(5 \notin B, 6 \notin B)$

$\emptyset \subseteq B$!!

Def A subset of B is PROPER if it doesn't contain all the elts of B.

i.e. $A \subseteq B \wedge B \nsubseteq A$.

Notes ① to prove $A = B$, show $A \subseteq B$ and $B \subseteq A$.

(Think: $x = y \iff (x \leq y$ and $y \leq x)$)

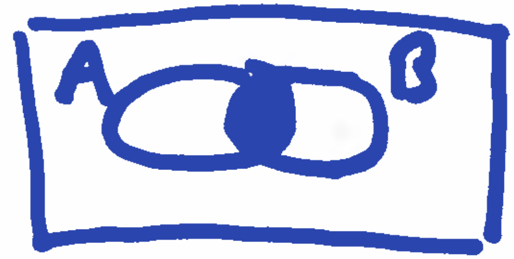
② Some books use $\subset$, $\subseteq$ for "proper", "proper or equal to." (Think: $<, \leq$) Most use $\subset$ for both. Ours uses $\subseteq$.

Forming New Sets from Old

Venn Diagram

Intersection

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$



Union

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

Set Difference

$$A - B = A \setminus B = \{x \mid x \in A \wedge x \notin B\}$$

Complement

$$\bar{A} = A^c = \{x \mid \sim(x \in A)\} = X \setminus A$$

$$= \{x \notin A\}$$ (X is universe set)

Ex In $\mathbb{N}$, let $A =$ even #'s, $B = \{1,2,3,\dots,10\}$

$A \cap B = \{2,4,6,8,10\}$

$A \cup B = \{1,2,3,\dots,10\} \cup \{12,14,16,\dots\} = A \cup \{1,3,5,7,9\}$

$\overline{A} = A^c =$ odds

$A \setminus B = \{12, 14, 16, \dots\}$

$B \setminus A = \{1,3,5,7,9\}$

$A \cup \emptyset = A$

$B \cap \emptyset = \emptyset$

Ex Prove: $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

like: $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$

Pf We must show $X \cap (Y \cup Z) \subset (X \cap Y) \cup (X \cap Z)$

and $(X \cap Y) \cup (X \cap Z) \subset X \cap (Y \cup Z)$.

(We show the first inclusion here)

Let $x \in X \cap (Y \cup Z)$. Thus $x \in X$ (*) **and** $x \in (Y \cup Z)$, which means $x \in Y$ or $x \in Z$.

(*): Here it's important that we **know** $x \in X$

If $x \in Y$, then $x \in X \cap Y$ since $x \in X$ as well.

Similarly, if $x \in Z$ then $x \in X \cap Z$.

Thus $x \in X \cap Y$ or $x \in X \cap Z$, which means

$x \in (X \cap Y) \cup (X \cap Z).$

Hence $X \cap (Y \cup Z) \subset (X \cap Y) \cup (X \cap Z).$

In other direction, assume

$x \in (X \cap Y) \cup (X \cap Z)$

. $\Rightarrow x \in (X \cap Y)$ or $x \in (X \cap Z)$

. if $x \in$ it means... and if $x \in$, ...

.

.

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. $x \in X$ and $x \in Y \cup Z$

Thus $x \in X \cap (Y \cup Z)$

Indexed Sets Often we use families of sets.

Ex $A_n = [-n, n]$, $n \in \mathbb{N}$ n = index
$\mathbb{N}$ = indexing set, set of indices.

$A_1 = [-1, 1]$ $A_{100} = [-100, 100]$
$A_2 = [-2, 2]$

We often use notation similar to

$$\sum_{n=1}^{5} a_n = a_1 + a_2 + a_3 + a_4 + a_5$$

when dealing with indexed sets.

$$\bigcup_{n=1}^{5} A_n = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 = A_5$$

$$\bigcup_{n=1}^{\infty} A_n = \mathbb{R} \qquad \bigcap_{n=2}^{\infty} A_n = [-2, 2]$$