§8.1 Infinite Series

Adding up infinitely many #'s is tricky. To wit:

$$
\begin{aligned}
0 &= 0 + 0 + 0 + 0 + \cdots \\
&= (1-1) + (1-1) + (1-1) + (1-1) + \cdots \\
&= 1 + (-1+1) + (-1+1) + (-1+1) + \cdots \\
&= 1 + 0 + 0 + 0 + \cdots \\
&= 1 + 0 \\
&= 1.
\end{aligned}
$$

Recall given (a_n), $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + \dots$

s_1 s_2 s_3 s_4

A sum of the terms in a sequence is a series; above we have an infinite series

When can we say an infinite series has a value? (i.e. equals a real number)

$\sum a_n$ has an associated seq. of partial (truncated) sum

$s_1 = a_1$

$s_2 = a_1 + a_2$

$s_3 = a_1 + a_2 + a_3$

$\vdots$

$s_n = a_1 + a_2 + \dots + a_n = \sum_{k=1}^{n} a_k$ = "n^{th} partial sum of $\sum a_n$."

If (and only if) $s_n \to s$ may we say

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots = s \in \mathbb{R}.$$

Otherwise the series diverges and does not equal a real #.

⚠ Warnings

① $a_1 + a_2 + a_3 + \cdots$ has no arithmetical value unless $\sum a_n$ converges. So $\underbrace{a_1 + a_2 + a_3 + \cdots}_{s.}$ is really $\underbrace{\lim (a_1 + a_2 + \cdots a_n)}_{\lim s_n}$

② Think of $a_1 + a_2 + a_3 + \cdots$ as one object. Don't apply laws of arithmetic to infinite sums. Don't rearrange, regroup, etc.

$\boxed{\text{Exercise } 8.1.16(3)}$

$\underline{\text{Ex}}$ In first example,

(on board) $1-1+1-1+1-\cdots$

has partial sums $(1,0,1,0,1,0,\ldots)$ which diverges.

$\underline{\text{Ex}}$ $\sum_{n=0}^{\infty} r^n = 1+r+r^2+r^3+r^4+\cdots$

$s_0 = 1$

$s_1 = 1+r$

$\vdots$ (by induction)

$s_n = 1+r+\cdots+r^n = \dfrac{1-r^{n+1}}{1-r} \longrightarrow \dfrac{1}{1-r}$ $\underline{\underline{\text{if}}}$ $|r|<1$.

More generally, $\sum_{n=0}^{\infty} ar^n = \dfrac{a}{1-r}$, $|r|<1$ (HW?)

<u>Ex</u>

(we start w/ $n=1$)

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots = \left(\sum_{n=0}^{\infty} \frac{1}{2^n}\right) - \left(\frac{1}{2}\right)^0 = 2-1=1.$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1-1/2} = \frac{1}{1/2} = 2.$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2}{3^n} = \sum_{n=0}^{\infty} 2\cdot\left(\frac{-1}{3}\right)^n = \frac{2}{1-(-1/3)} = \frac{2}{1+3} = \frac{2}{4/3} = \frac{6}{4}$$

$$= \frac{3}{2}.$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = 1+\frac{3}{2}+\frac{9}{4}+\frac{27}{8}+\cdots = \infty \quad b/c \quad r=\frac{3}{2}>1$$

$$\neq \frac{1}{1-3/2} = \frac{1}{-1/2} = -2.$$