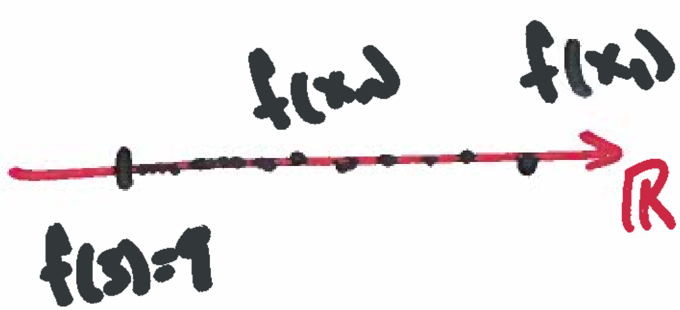
Aside: Writing Project Help

<u>Def</u> $f: D \subseteq \mathbb{R} \to \mathbb{R}$ continuous at $c \in D$ if, for every sequence (x_n) in D which converges to c, the seq. $(f(x_n))$ converges to $f(c)$.

<u>Ex</u> $f(x) = x^2$ is continuous at $x=3$.

if $x_n \to 3$, then

$$f(x_n) = x_n^2 \to f(3) = 9.$$



Reminds us of

$$\lim_{x \to a} f(x) = f(a).$$

Thm 1 (lite version) Let $f,g: D \to \mathbb{R}$ be continuous at $c \in D$. Then $f+g$ and fg are continuous at c.

Pf amounts to citing limit laws in Thm 4.2.1.

$f+g$ If f, g are continuous at $x=c$ and $x_n \to c$, then $f(x_n) \to f(c)$
$g(x_n) \to g(c)$.

To show $f+g$ is continuous, must show

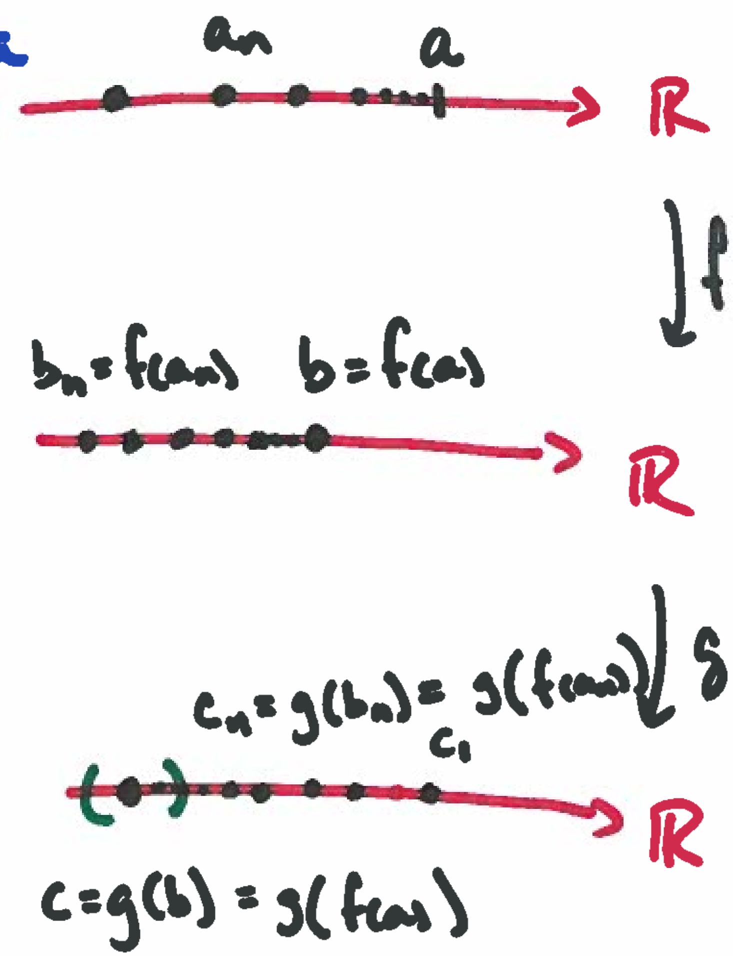
$$f(x_n)+g(x_n) \to f(c)+g(c).$$

Thm 4.2.1(a) says $\lim[f(x_n)+g(x_n)]$

$$= \lim f(x_n) + \lim g(x_n) = f(c) + g(c)$$

Thm 2 (lite) f, g continuous $\Rightarrow g \circ f$ continuous.

To prove $g \circ f$ is continuous at $x = a$, let $a_n \to a$ and $\varepsilon > 0$. Need to show $\exists$ N s.t. $n > N$, $\underline{|g \circ f(a_n) - g \circ f(a)| < \varepsilon}$.



Define $b_n = f(a_n)$; since f is cont, we know $b_n \to b = f(a)$.

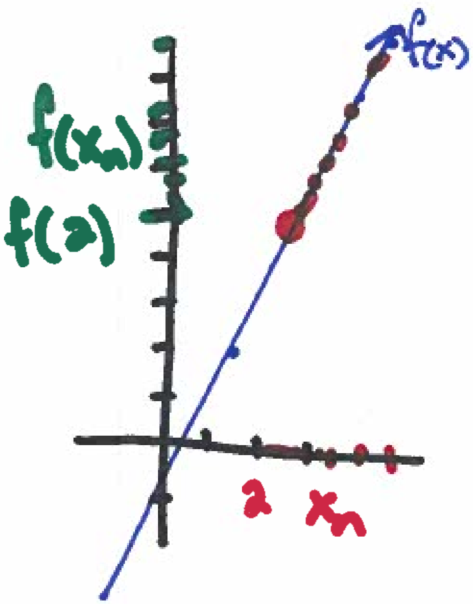
$c_n = g(b_n) = g(f(a_n)) \to c$ b/c g is continuous.

Need to show $\exists$ N s.t. $n > N \Rightarrow |c_n - c| < \varepsilon$; such an N exists b/c g is a continuous fn, and c_n is image of conv. seq, b_n.

Recall our Writing Project Def^n:

$f: D \subseteq \mathbb{R} \to \mathbb{R}$ is continuous at $c \in D$ if, for every convergent sequence in D, $x_n \to c \Rightarrow f(x_n) \to f(c)$.

Ex $f(x) = 3x - 1$ is continuous at $x = 2$



Pf Let $x_n \to 2$; we need to show $f(x_n) \to f(2)$; for any $\varepsilon > 0$ must find N s.t. $n > N \Rightarrow |f(x_n) - f(2)| < \varepsilon$

Given $\varepsilon > 0$, choose N large enough s.t. $|x_n - 2| < \varepsilon/3$, whenever $n > N$.

Then $|f(x_n) - f(2)| = |3x_n - 1 - 5| = |3x_n - 6| = 3|x_n - 2|$

$$< 3 \cdot \varepsilon/3 = \varepsilon.$$

<u>Recall</u>: You may cite Thm 4.2.1

Alt. explanation of $f(x) = 3x-1$ cont. at $x=2$:

Want to show: given any $x_n \to 2$, the seq. $f(x_n) \to f(2) = 5$.

By Thm 4.2.1, given $x_n \to 2$,

$$\begin{aligned}
\lim \underbrace{3x_n - 1}_{f(x_n)} &= \left(\lim 3x_n\right) - 1 \\
&= 3\left(\lim x_n\right) - 1 \\
&= 3(2) - 1 = 6 - 1 = 5 = f(2) \checkmark
\end{aligned}$$

(Provided these limits exist, etc)