From our previous "abstract" example...

$R = \{(a,a), (b,b), (c,c), (d,d), (a,b), (c,b), (b,a), (b,c), (a,c), (c,a)\}$

aRa dRd aRb ....

Equivalence classes:

$E_a = \{a, b, c\}$

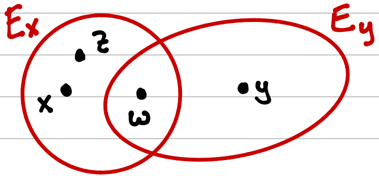
$E_d = \{d\}$

So R "partitions" set $S = \{a,b,c,d\}$ into two disjoint subsets.

Proposition Different Equiv. classes are disjoint.

If E_x and E_y are equiv. classes for eq rel'n $\sim$, then $E_x \cap E_y = \emptyset$ or $E_x = E_y$.

Pf: Let $x, y \in S$ with eq rel'n $\sim$.



⚠ This diagram in book is misleading!

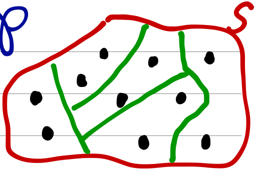
If $E_x \cap E_y = \emptyset$, we're done!

Else $\exists\, w \in E_x \cap E_y$. We can show $E_x \subset E_y$ and $E_y \subset E_x$ which means $E_x = E_y$.

Suppose $z \in E_x$. Then $z \sim x \sim w \sim y$ (b/c $w \in E_x$ and $w \in E_y$).
By transitivity, $z \sim y \Rightarrow z \in E_y \Rightarrow E_x \subset E_y$.

($E_y \subset E_x$ similar)

Def A partition of a set S is a collection $\mathcal{P}$ of non-empty subsets of S such that



(a) $\forall x \in S,\ \exists A \in \mathcal{P}$ s.t. $x \in A$

(b) $\forall A, B \in \mathcal{P}$, either $A \cap B = \emptyset$ or $A = B$.

Ex $S = \{\underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6}, \underline{7}, \underline{8}, \underline{9}, \underline{10}\}$

$A = \{1, 2, 3\}$ $B = \{4, 6, 8, 10\}$ $C = \{5\}$, $D = \{7, 9\}$

$\Rightarrow \mathcal{P} = \{A, B, C, D\}$ partition S.

Ex Assigning 6^{th} graders to soccer teams

(a) says everybody is on a team

(b) says no student on two different teams.

Key idea Equivalence Classes Form a Partition

Any elt x in a set S with eq reln $\sim$ belongs to an eq class (E_x), and the classes are disjoint (Prop'n)

Ex $S=$ UMN Students, $xRy \Leftrightarrow x,y$ born in same year

R is an equiv. rel'n (you check)

$\vdots$

$E_{1993} = \{$ — " — $1993\}$

$E_{1994} = \{$people born in $1994\}$

$E_{1995} = \{$ — " — $1995\}$

$\vdots$

$\mathcal{P} = \{E_{1900}, \ldots, \ldots, E_{2016}\}$ form a part'n of student body.

Thm 2.2.17

(a) If S has eq rel'n R, the eq classes form a part'n of S.

(b) If $\mathcal{P}$ is a partion of S, the relation

$xRy \iff x, y$ are in same set of part'n

is an equivalence relation.

Ex