§4.1 Sequences and Convergence

Why study sequences?

They're a basic math'l object. Much of calculus can be described in terms of sequences:
Continuous fns, limits, (hence deriv's / integrals)

From a learning standpoint:

$\lim\limits_{n\to\infty} a_n$ is "easier" than $\lim\limits_{x\to a} f(x)$.

Informally, a seq. is a list of (real) numbers:

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

Formal Def A sequence in $\mathbb{R}$ is a **function** $f: \mathbb{N} \to \mathbb{R}$

(So $f(i)$ is i^{th} # in "list".)

Above: $f(n) = \frac{1}{n}$; $f(1) = 1$, $f(2) = \frac{1}{2}$, $f(3) = \frac{1}{3}$, ...

Usually, we avoid fn notation and use subscripts:

$a_1 = f(1)$, $a_2 = f(2)$, $a_3 = f(3)$

Above $(a_n) = (1/n)$

$(a_n) = (1, 1/2, 1/3, 1/4, \ldots)$

(a_n) = the sequence $(a_1, a_2, a_3, \ldots)$

$\{a_n\}$ = set of #'s in sequence.

Ex $a_n = \sin(n \cdot \pi/2)$ $\quad (a_n) = (1, 0, -1, 0, 1, 0, -1, \ldots)$

$\{a_n\} = \{1, 0, -1\}$.

many, many books use $\{a_n\}$ for what our book calls (a_n).

Ways to Define a Sequence

① Give a formula for n^{th} term

$a_n = \frac{1}{n}$ OR $(a_n) = \left(\frac{1}{n}\right) = \left(\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right)$

② Give 1^{st} term(s) and a recursive formula.

$a_1 = 1$, $a_n = 2a_{n-1} + 3$ gives $(1, 5, 13, 29, \ldots\ldots)$

$a_1 = a_2 = 1$ $\quad a_n = a_{n-1} + a_{n-2}$ $\quad (1, 1, 2, 3, 5, 8, 13, 21, \ldots)$

③ List enough terms to establish a pattern.

$(a_n) = (1, 4, 9, 16, 25, 36, \ldots)$ (square #'s, $a_n = n^2$)

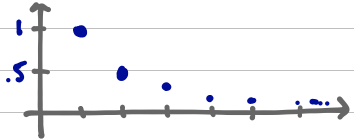
⚠ Risky. What if somebody doesn't see pattern?
Or a different pattern?

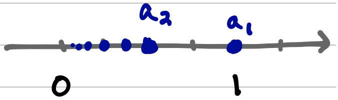
$(b_n) = (0, 7, 26, \ldots)$ $\qquad [b_n = n^3 - 1]$

④ Graphically [$\exists$ at least two common ways]

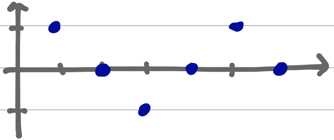
→ Helpful, but not rigorous ←

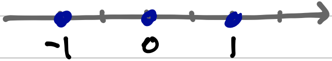
$a_n = \frac{1}{n}$





$b_n = \sin \frac{\pi}{2} \cdot n$





$c_n = \left\lfloor \frac{2}{n} \right\rfloor$