§8.1 Infinite Sums

Adding up infinitely many #'s is tricky. To wit:

$$0 = 0 + 0 + 0 + 0 + 0 + \cdots$$

$$= (1-1) + (1-1) + (1-1) + (1-1) + (1-1) + \cdots$$

$$= 1 + (-1+1) + (-1+1) + (-1+1) + (-1+1) + \cdots$$

$$= 1 + 0 + 0 + 0 + 0 + 0 + \cdots$$

$$= 1 + 0$$

$$= 1$$

Recall, given (a_n), $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + \cdots$

$s_1 \quad s_2 \quad s_3 \quad s_4$

A sum of the terms in a sequence is a series; above we have an infinite series

When can we say an infinite series has a value? (i.e. equals a real #.)

$\sum a_n$ has an associated seq of partial (truncated) sums

$s_1 = a_1$

$s_2 = a_1 + a_2$

$\vdots$

$s_n = a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$ = "n^{th} partial sum of $\sum a_n$"

If (and only if) $s_n \to s$ may we say

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots = s \in \mathbb{R}$$

Otherwise the series diverges and does not equal a #.

⚠ Warnings

① $a_1 + a_2 + a_3 + \cdots$ has no arithmetical value unless $\sum a_n$ converges. So $a_1 + a_2 + a_3 + \cdots$ means $\lim \underbrace{(a_1 + a_2 + \cdots + a_n)}_{\lim s_n}$

② Think of $a_1 + a_2 + a_3 + \cdots$ as one object. Don't apply laws of arithmetic to infinite sums. Don't rearrange, regroup, etc.