

The following is a *non-comprehensive* list of solutions to the computational problems on the homework. For some problems there is a sketch of a solution. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work. Feel free to talk to us about any of the writing intensive problems.

Please let us know if you spot any typos and we'll update things as soon as possible.

1.3 our answers may vary according to whether you know and use the antonyms to some of the terms being negated. For example, “not singular” is also referred to as “nonsingular.”

- (a) The 3×3 identity matrix is not singular.
- (b) The function $f(x) = \sin x$ is not bounded on \mathbb{R} .
- (c) f is not linear or g is not linear.
- (d) Six is not prime and seven is even.
- (e) x is in D and $f(x) \geq 5$. (The negation of an implication **IS NOT** an implication!)
- (f) (a_n) is monotone and bounded and (a_n) diverges.
- (g) f is injective and S is both infinite and not denumerable.

- 1.10 (a) False, because the second statement (5 is even) is false.
- (b) False, because both statements ($3 + 4 = 5$, $4 + 5 = 6$) are false.
- (c) True, because the second statement (6 is not prime) is true.
- (d) False, because the antecedent ($4 + 4 = 8$) is true but the consequent (9 is prime) is false.
- (e) True, because the antecedent (6 is prime) is false, so the implication is true regardless of the consequent.
- (j) True. The double negatives make this difficult to read; in symbols we have

$$\sim [\sim (5 \text{ is prime}) \wedge (3 \text{ is odd})]$$

Using De Morgan's Law, $\sim [p \wedge q] \Leftrightarrow (\sim p) \vee (\sim q)$, this is equivalent to

$$(5 \text{ is prime}) \vee (3 \text{ is even})$$

This disjunction is true, because the first statement is true.

- 1.12 (a) $\sim m \wedge n$
- (b) $(\sim m) \wedge (\sim n)$. This is also $\sim (m \vee n)$
- (c) $n \Rightarrow m$. The statement “ q only if p ” is equivalent to $q \Rightarrow p$. One way to see that is to observe that the only way “ q only if p ” is false is if q is true and p is false. If you look at the truth table for an implication, this exactly describes the truth values of $n \Rightarrow m$.
- (d) $m \Rightarrow \sim n$. The statement “ q provided that p ” is equivalent to $p \Rightarrow q$. Informally, if you think of “if p then q ” as “if (hypothesis is true), then (conclusion),” you can see that equivalence. In “ q provided that p ,” we check if p is true (hypothesis) and, if so, q would be true (conclusion). [That's assuming the implication as a whole is true.]
- (e) $\sim [m \wedge n]$