The following is a *non-comprehensive* list of solutions to the computational problems on the homework. For some problems there is a sketch of a solution. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work. Feel free to talk to us about any of the writing intensive problems.

Please let us know if you spot any typos and we'll update things as soon as possible. Update: 12/8/16: fixed a typo in the solution to 4.1 #4.]

4.1 #4 There is no single answer to this question; your value of m will depend on your choice of k. One thing is clear: for

$$6n^3 + 17n \leq kn^3$$

to hold for a natural number n, the constant k must be at least 7. If you plug $k = 7$ into the inequality and solve for n, you'll find $n \geq \sqrt{17} \approx 4.123$, so if you set $m = 5$ you can be sure the inequality will hold for $n \geq m$. For $k = 8$, 9 or 10, setting $m = 3$ would work.

4.1 #6(a) Let $\varepsilon > 0$. We need to find an N such that $n > N$ implies $\left|\frac{k}{n} - 0\right| = |k/n| < \varepsilon$. This last inequality is equivalent to $n > |k|/\varepsilon$, so choosing $N = |k|/\varepsilon$ will suffice. Ask us if you need help figuring out how to write out a formal proof of the limit.

4.1 #9 (a) This is true. Suppose $s_n \to s$, and let's prove $|s_n| \to |s|$. Let $\varepsilon > 0$, in which case there exists N such that $n > N$ implies $|s - s_n| < \varepsilon$. The key tool for this problem is the variant of the triangle inequality you proved in Exercise 3.2 #6(a): $||x| - |y|| \leq |x - y|$. Applying that here for those same values of n, we have:

$$||s_n| - |s|| \leq |s_n - s| < \varepsilon$$

Thus $|s_n| \to |s|$.

(b) This is false. Very, very false. A simple example would be $s_n = (-1)^n$, in which case $(|s_n|) = (1, 1, 1, \ldots)$ certainly converges to 1, but s_n does not converge (as proven in lecture).

(c) This is true, and follows from the algebraic equivalence of $|s_n - 0| < \varepsilon$ and $||s_n| - 0| < \varepsilon$. (Hint: the left hand sides of the inequalities are the same!)

4.1 #10 (a) One possible answer would be $a_n = \left(1 + \dfrac{1}{n}\right)^n$, which converges to e. (You don't need to prove it converges for this particular problem; you might recall that this is actually the definition of e in some precalculus or calculus textbooks.) Each number in this sequences is a power of a rational number, and hence is rational. Its limit, e, is not rational number.

(b) One possible answer would be $b_n = \dfrac{\pi}{n}$. Then each b_n is irrational, but $b_n \to 0 \in \mathbb{Q}$.

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