

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics. (5 Points)

- (1) (6 Points) Prove: if x is rational and y is rational, then xy is rational.

PF Suppose x and y are rational, so $x = \frac{a}{b}$ and $y = \frac{c}{d}$ for integers a, b, c, d

with $b \neq 0$ and $d \neq 0$. Then

$$xy = \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Both ac and bd are integers, and $bd \neq 0$ because both b and d are nonzero.

Thus xy is rational.

- (2) (9 Points) Prove: if pq is odd, then p is odd and q is odd.

PF We'll prove the equivalent contrapositive statement,

if p is even or q is even, then pq is even.

Suppose p is even, so $p = 2k$ for some integer k . Then $pq = 2k \cdot q = 2(kq)$, which is even.

Similarly, if $q = 2l$ for some $l \in \mathbb{Z}$ then $pq = p \cdot 2l = 2(pl)$, which is even.

Hence the contrapositive statement holds.

+3 (some implicit argument for second case is ok)