Math 3283W Fall 2016 - Writing Quiz #3
Thursday 9/29/16 - 15 minutes **Name:**

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics. (5 Points)

(1) (9 Points) Prove: if x is rational and y is irrational, then $x + y$ is irrational.

+1 Method

Pf: Proof by contradiction. Assume not,

so x is rational, y is irrational, and $x+y$ is rational. +2 Correct negation of implication

Then $x = \frac{a}{b}$ and $x+y = \frac{c}{d}$ for integers a, b, c, d with $b \neq 0$ and $d \neq 0$. +2

Hence $y = (x+y) - x = \frac{c}{d} - \frac{a}{b} = \frac{bc-ad}{bd}$. +2 The numerator and denominator

are integers, and $bd \neq 0$ because b and d are nonzero. +1 Thus y is rational, +1 which

contradicts our assumption.

(Therefore the assumption was wrong and the original statement is true.)

(2) (6 Points) Let A, B, and C be subsets of a universal set U. Prove $A \setminus (B \cup C) \subseteq (A \setminus B) \cap (A \setminus C)$.

Pf: Let $x \in A \setminus (B \cup C)$. +1 This means $x \in A$ and $x \notin B \cup C$. +1 In other words,

$x \in A$ and x is in neither B nor C. +1

Since $x \in A$ and $x \notin B$, we have $x \in A \setminus B$. +1 Similarly, because $x \in A$ and $x \notin C$,

x is in $A \setminus C$; +1 because x is in both sets, we know

$$x \in (A \setminus B) \cap (A \setminus C). \text{ +1}$$

Thus any element of $A \setminus (B \cup C)$ is in $(A \setminus B) \cap (A \setminus C)$, which means

$$A \setminus (B \cup C) \subseteq (A \setminus B) \cap (A \setminus C).$$