Math 3283W Fall 2016 - Writing Quiz #5

Thursday 10/20/16 - 20 minutes **Name:**

Remember: your work on in the "writing" portion of this quiz will be graded on the quality of your writing and explanation as well as the validity of the mathematics. (5 Points)

Definitions. This portion of your quiz will be graded for mathematical correctness only.

(1) (3 Points) Complete the definition: two sets A and B are *equinumerous* if...

$\exists$ bijection $f: A \to B$

(2) (3 Points) Define what it means for a set A to be *finite*, *denumerable*, or *countable*.

A finite if A is equinumerous with $I_n = \{1, 2, \dots, n\}$, some n

A denumerable if A is equinumerous with $\mathbb{N}$.

A is countable if it is finite or denumerable.

Writing. Recall that a function is a *bijection* if it is both injective and surjective.

(3) (9 Points) Prove that the intervals $(0, 1)$ and $(5, 9)$ are equinumerous. (You must prove that any functions you define have the required properties.)

Define $f: (0,1) \to (5,9)$, $f(x) = 4x+5$. Then f is injective, because $f(x) = f(x_2)$ implies

$$4x_1+5 = 4x_2+5$$
$$4x_1 = 4x_2$$
$$x_1 = x_2.$$

The function f is also surjective. Let $y \in (5,9)$. We can find $x \in (0,1)$ such that $f(x) = y$ as follows:

$$4x+5 = y$$
$$4x = y-5$$
$$x = \frac{y-5}{4}.$$

Note that if $5 < y < 9$, then $0 < y-5 < 4$ and $0 < \frac{y-5}{4} < 1$ so $x \in (0,1)$, as desired.