Math 3283W Fall 2016 - Writing Quiz #6

Thursday 10/27/16 - 20 minutes **Name:**

Remember: your work on in the "writing" portion of this quiz will be graded on the quality of your writing and explanation as well as the validity of the mathematics. (5 Points)

Writing. Recall that a function is a *bijection* if it is both injective and surjective. If you define a function which you claim is injective, surjective, or bijective, you must prove that assertion.

(1) (6 Points) Prove that $\mathbb{N}$ is equinumerous with a proper subset of itself.

Let $S = \{2, 3, 4, \dots\}$ +1 for proper denum. subset and define $f: \mathbb{N} \to S$, $f(n) = n+1$. +1 function

The function f is injective, because if $n \neq m$, $f(n) = n+1 \neq m+1 = f(m)$. +1 inj.

The function f is surjective, because if $m \in S$, then $m-1 \in \mathbb{N}$ and $f(m-1) = m$. +1 surj.

Thus f is bijective and $\mathbb{N} \sim S$. +2 knowing we need a bij'n.

(2) (9 Points) Prove that $\mathbb{N} \times \mathbb{N}$ is denumerable.

Method 1

Mirror proof that $\mathbb{Q}^+$ is denumerable:

$\mathbb{N} \times \mathbb{N}$ as grid: +3

$(1,1), (1,2), (1,3), \dots$
$(2,1), (2,2), (2,3), \dots$
$(3,1), (3,2), (3,3), \dots$
$\vdots$

Zig-zag path +2

$f(n) = n^{\text{th}}$ ordered pair on path +2

Expl'n that f inj, surj. +2

Method 2 (Using Thm 2.4.10)

We must construct an injection $\mathbb{N} \times \mathbb{N} \to \mathbb{N}$. +3

Define $f: \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$, $(m,n) \longmapsto 3^m 5^n$ +3 for a fn def$^{\underline{n}}$.

This is an injection, because the fund. thm. of arithmetic (e.g. prime factorization) tells us that if

$$3^m 5^n = 3^j 5^k,$$

then $m = j$ and $n = k$, i.e. $(m,n) = (j,k)$.

+3 for explanation of why f is injective.