**Math 3283W Fall 2016 - Writing Quiz #8**
**Thursday 11/17/16 - 20 minutes** **Name:** Key

Remember: your work on in the "writing" portion of this quiz will be graded on the quality of your writing and explanation as well as the validity of the mathematics. (5 Points)

**Definitions.** This portion of your quiz will be graded for mathematical correctness only.

(1) (4 Points) Complete the definition: $s_n$ converges to $s$, written $s_n \to s$ or $\lim_{n\to\infty} s_n = s$, if...
*(You may use symbols for logic quantifiers in your answer.)*

$\forall\, \varepsilon>0\ \exists\ N$ s.t. $\forall n,\ n>N \Rightarrow |s_n - s| < \varepsilon$

not necessary, but could be helpful when negating in (b)

(2) (2 Points) Negate your answer to (1) to write a definition of: $s_n$ does *not* converge to $s$ if...
*(Do not just write "not" in front of your answer to (1).)*

$\exists\, \varepsilon>0$ s.t. $\forall N, \exists n$ s.t. $n>N$ and $|s_n - s| \geq \varepsilon$.

(3) (1 Point) Complete the definition: $s_n$ diverges if...

$\forall s, \exists\, \varepsilon>0$ s.t. $\forall N, \exists n$ s.t. $n>N$ and $|s_n - s| \geq \varepsilon$.

[ i.e. $\forall s$, $s_n$ does not converge to $s$ ]

**Writing.** This portion of your quiz will be graded for both writing and correctness.

(4) (8 Points) Use the definition of limit in our class to prove $s_n = \dfrac{3n+1}{n+2}$ converges to 3. You may use the back side of this paper to do algebra / scratchwork / preparation. Your proof on this page should be self contained – in particular, it should show any algebra steps, even if you have worked them out on the other side of this sheet.

Think/alg/prep
(not graded unless it's necessary to look at algebra to grant partial credit.)

$\left|\frac{3n+1}{n+2} - 3\right| < \varepsilon$

$\left|\frac{3n+1-3n-6}{n+2}\right| < \varepsilon$

$\left|\frac{-5}{n+2}\right| < \varepsilon$

$\frac{5}{n+2} < \varepsilon$

$\frac{5}{\varepsilon} - 2 = n$

(Suggests $N = 5/\varepsilon - 2$)

Proof: Let $\varepsilon>0$, and choose $N = \frac{5}{\varepsilon} - 2$. Then $n>N$ implies

+2 appropriate choice of $N$

$$\left|\frac{3n+1}{n+2} - 3\right| = \left|\frac{3n+1-3n-6}{n+2}\right| = \left|\frac{-5}{n+2}\right| = \frac{5}{n+2} < \frac{5}{N+2} = \frac{5}{5/\varepsilon} = \varepsilon.$$

+2 showing necessary algebra (within proof)

So a proof with an $N$ that doesn't work could earn +4 for structure, and +1 or +2 partial credit pts for alg which went into finding $N$, or the algebra in $|(\ ) - 3| < \varepsilon$.

+4 For correct structure of formal pf: $\varepsilon$ first, then $N$, then $n>N$, then $|\ | < \varepsilon$.

Thus $\left|\frac{3n+1}{n+2} - 3\right| < \varepsilon$, as desired.