

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

- (1) (6 Points) Prove: the square of an odd integer is odd.

Let n be an odd integer, so $n=2k+1$ for some integer k . Then

$$\begin{aligned}n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1,\end{aligned}$$

which has the form of an odd integer. Hence n^2 is odd.

Math 15/15 Overall: 20/20
Writing 5/5

- (2) (9 Points) Recall the following definition: x is a *rational* number if and only if it can be written in the form $x = \frac{a}{b}$, where a and b are integers and $b \neq 0$. Prove: the sum of two rational numbers is rational.

Let x and y be rational, so $x = \frac{a}{b}$ and $y = \frac{c}{d}$ for integers a, b, c and d with $b \neq 0$ and $d \neq 0$. Then

$$\begin{aligned}x + y &= \frac{a}{b} + \frac{c}{d} \\ &= \frac{ad+bc}{bd}\end{aligned}$$

Note that $ad+bc$ and bd are both integers, and $bd \neq 0$ because both b and d are nonzero. Thus $x+y$ is rational.

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- (1) (6 Points) Prove: the square of an odd integer is odd.

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1,$$

Overall, the math in both parts is mostly correct (what's k ?) for 14/15
The writing score would be 0/5, or at most 1/5.

No explanation at all in (1), not even of why $n=2k+1$. Similar issues in (2), plus an excessive amount of unneeded work.

- (2) (9 Points) Recall the following definition: x is a *rational* number if and only if it can be written in the form $x = \frac{a}{b}$, where a and b are integers and $b \neq 0$. Prove: the sum of two rational numbers is rational.

$$\begin{aligned} \frac{a}{b} + \frac{c}{d} &= \frac{a}{b} \cdot \frac{bd}{bd} + \frac{c}{d} \cdot \frac{bd}{bd} \\ &= \frac{abd}{b^2d} \cdot \frac{d}{d} + \frac{bcd}{bd^2} \cdot \frac{b}{b} \\ &= \frac{abd^2}{b^2d^2} + \frac{b^2cd}{b^2d^2} \\ &= \frac{abd^2 + b^2cd}{b^2d^2} \\ &= \frac{abd^2 + b^2d^2 - b^2d^2 + b^2cd}{b^2d^2} \\ &= \frac{d^2b(a+b) - b^2d(d+c)}{b^2d^2} \\ &= \frac{d(a+b) - b(d+c)}{bd} \end{aligned}$$

Overall: 15/20

Now let $z = d(a+b) - b(d+c)$ and $w = bd$. Note $w \neq 0$ because $b, d \neq 0$. Then z/w is rational!

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- (1) (6 Points) Prove: the square of an odd integer is odd.

Let n be an odd integer, so $n=2k+1$ for some integer k . Then

$$n^2 = (2k+1)^2$$

$$= 4k^2 + 2k + 2 \leftarrow \text{major math error, but the writing is well organized and consistent.}$$

$$= 2(2k^2 + k + 1),$$

which is even, hence the problem is wrong.

3/6 for math

- (2) (9 Points) Recall the following definition: x is a *rational* number if and only if it can be written in the form $x = \frac{a}{b}$, where a and b are integers and $b \neq 0$. Prove: the sum of two rational numbers is rational.

Let $x = \frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$ and $y = \frac{c}{d}$ where $c, d \in \mathbb{Z}$ and $d \neq 0$ and

then add them together: $x+y = \frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$ which is clearly in the form n/m

where $n = a+c$ and $m = b+d$ and these are both integers, so the sum of two

rational numbers is rational.

Major math error in addition, and didn't check that $b+d \neq 0$. $4/9$ would be reasonable.

The writing is ok in (1) but an issue in (2).

It's one long run-on sentence, and one large block of text, which makes it difficult to read.

Some justification is also lacking.

Overall Score: 10/20

Averaging a 5/5 on (1) and 2/5 on (2) would give 3.5/5 overall for writing, probably rounded down because of extra weight of (2).

Math 3283W - Sample Writing Quiz

15 minutes

Name: _____

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

- (1) (6 Points) Prove: the square of an odd integer is odd.

$$1^2 = 1 \checkmark$$

odd odd

$$5^2 = 25 \checkmark$$

odd odd

$$9^2 = 81$$

odd odd

$$3^2 = 9 \checkmark$$

odd odd

$$7^2 = 49 \checkmark$$

odd even
odd

$$11^2 = \text{odd}$$

Math: at most 1/6. The work is relevant to problem stmt, but checking examples won't lead to a proof.

- (2) (9 Points) Recall the following definition: x is a *rational* number if and only if it can be written in the form $x = \frac{a}{b}$, where a and b are integers and $b \neq 0$. Prove: the sum of two rational numbers is rational.

Let x and y be rational numbers. Then $x = \frac{a}{b}$, $y = \frac{c}{d}$ where a, b, c, d are integers and $b \neq 0 \neq d$.

$$\text{Then } x+y = \frac{a}{b} + \frac{c}{d}.$$

Because the sum of two fractions is a fraction, $x+y$ must equal

$\frac{f}{g}$ for two integers f and g with $g \neq 0$.

So $x+y$ is rational.

math: at most 2/9. Definitions are used, but no progress made towards a proof

Overall: 4/20

writing: no points for (1) - there's no writing! In (2) there is writing, but it's not well organized and amounts to restating the problem. 1/5 overall