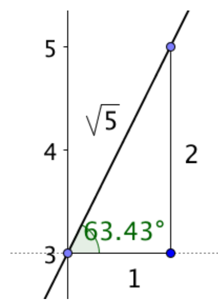
The following is a non-comprehensive list of solutions to homework problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

3.12, 3.13: The translation in these problems is easy to write a formula for: $\mathcal{T}(X) = X + (-3, -6)$ or, if you prefer, $\mathcal{T}(x, y) = (x - 3, y - 6)$. The equation for the reflection is trickier. We know $\langle(2, -1), (x, y)\rangle = -3$ is equivalent to $2x - y = -3$, or $y = 2x + 3$. So the slope of the mirror is 2, and hence the angle it forms with a horizontal line is $\theta = \arctan 2 \approx 63.4°$. The y-intercept form of the line makes it clear that $(0, 3)$ is on the line. Hence a matrix formula for the reflection across this line is:

$$\mathcal{M}(X) = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \left(\begin{bmatrix} X \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Here's a picture of the line:



Using the triangle in the picture, we see that

$$\cos 2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta = \frac{1}{5} - \frac{4}{5} = -\frac{3}{5}$$
$$\sin 2\theta = 2\cos\theta\sin\theta = \frac{4}{5}$$

Hence our formula for the reflection becomes

$$\mathcal{M}(X) = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left(\begin{bmatrix} X \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Now consider the two compositions (check the parentheses carefully to make sure you see the differences!):

$$\mathcal{T} \circ \mathcal{M}(X) = \left(\begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left(\begin{bmatrix} X \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} -3 \\ -6 \end{bmatrix}$$
$$\mathcal{M} \circ \mathcal{T}(X) = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left(\left(\begin{bmatrix} X \end{bmatrix} + \begin{bmatrix} -3 \\ -6 \end{bmatrix} \right) - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

If you distribute across the parenthesis, multiply the constant vectors by the matrix, and collect terms, you'll find that these are both (!) equal to

$$\begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \begin{bmatrix} X \end{bmatrix} + \begin{bmatrix} -27/5 \\ -24/5 \end{bmatrix}$$

So with these particular choices, it doesn't matter if you do the reflection first and then the translation, or vice versa; your answers to #6 and #7 are the same. (Why does it turn out that way? Hint:

$(-3,-6) = -3(1,2)$ could serve as a direction indicator for the line, so the composition in either order gives the same glide reflection!)

3.16: An involution is a function which is its own inverse. Given the formula $\mathcal{C}_C(X) = -X + 2C$ for a central inversion about a point C, we compute:

$$\mathcal{C}_C\left(\mathcal{C}_C(X)\right) = -(-X+2C) + 2C = (X-2C) + 2C = X$$

3.17: In general, as we've discussed in class, a fixed point of a function $f(x)$ is an input x for which $f(x) = x$ – i.e. the function "fixes" the input, or leaves it unchanged. So we just set up that equation with the formula for a central inversion:

$$\begin{aligned} \mathcal{C}_C(X) &= X \\ -X + 2C &= X \\ 2C &= 2X \\ C &= X \end{aligned}$$

which means that the only X fixed by a central inversion about a point C is C itself.

3.32(i,ii,iv): Refer to the illustrations below; ask me if you had difficulty computing any of the images of the points A, B and C under these isometries.