In class we've learned about the centroid, orthocenter, incenter and circumcenter of a triangle. It turns out these aren't the only "centers" and other points that you can construct from a triangle. There are lots of them. *Lots* of them. http://mathworld.wolfram.com/TriangleCenter.html lists twenty such points, but this Encyclopedia of Triangle Centers lists over 7000. The vast majority of have no specific applications or uses, and are unknown to all but a handful of people.

There are very cool and surprising relationships between some of these points. Sometimes they're evidence of deep, dark, and mysterious connections between various concepts. In other cases, well... it might just be a coincidence. Spend enough time looking at 7000+ points and eventually you'll find something surprising, but that doesn't mean you've unlocked some fundamental concept of geometry that has gone unnoticed for thousands of years. (But if it turns out you have, you can give me a shout-out in your acceptance speeches when you win major awards.)

In this lab you'll explore a relationship involving the midpoints of the sides of $\triangle ABC$ and its altitudes, and construct the so-called *Nine-Point Circle* of the triangle, sometimes called the *Feuerbach Circle*. (And, of course, sometimes called the *Euler Circle*.[1])

GEOGEBRA CONSTRUCTION

(1) Open a new GeoGebra window. Go to the View menu and turn off the axes, so the drawing pad is blank. I'd also recommend going to the Options menu, choose Labeling, and make sure "New Points Only" is selected to avoid having GeoGebra display a label for every single line segment you create.

(2) Choose the *Polygon* tool (it looks like a triangle) and click three times (and once more at your first point) to create $\triangle ABC$. Make sure each vertex angle is acute.

(3) Use the *Midpoint* tool to add D, E and F, the midpoints of the sides opposite your vertices. You could also do this with the Input field: enter `D = Midpoint[A,B]`, and so on.

(4) Use the *Perpendicular Line* tool to select a vertex and the opposite side. Repeat for the other vertices, so you now have three altitutdes. You want to find the feet of the altitudes. It's tempting to use the *Intersection* tool to click on an altitude and the segment it crosses, but **do not do this**. It will only give you the intersection of the altitude with the line **segment**, and if the altitude is outside the triangle it won't intersect with the segment at all! Instead, use the following commands in the Input field to get the intersection of the altitudes with the **lines** through the opposite vertices. Assuming d, e and f are the altitudes from A, B and C, respectively:

$$K = \text{Intersect}[d, \text{line}[B,C]]$$
$$L = \text{Intersect}[e, \text{Line}[A,C]]$$

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[1] I haven't gone back to verify this from the original sources, but according to the usual story, Feuerbach proved that one circle goes through *six* of the points described below, and Euler came along a few years later and showed the same circle passes through the other three points. Normally, we'd say "Euler proved something cool about Feuerbach's Circle." Objects are named by (or in honor of) the people who first discovered them. If I figured out something new about the Platonic Solids, they wouldn't be renamed the "Rogness Solids," for example. But Euler is so omnipresent in mathematics that his name has a way of getting attached to everything.

To be fair, I've also seen the story told the other way: that *Euler* discovered the six-point-circle, and Feuerbach proved it passes through the other three points. So perhaps history got it correct. If nothing else, be sure to take every mathematical urban legend with a grain of salt!

(5) Use the *Intersect* tool to find the orthocenter. Rename it H.

(6) Use the *Midpoint* tool to find the midpoints of the segments from the vertices to the orthocenter. Rename them P, Q and R, where $P = (A + H)/2$, $Q = (B + H)/2$, and $R = (C + H)/2$.

(7) Use the *Circle* tool to create a circle which goes through the midpoints of the sides of the triangles. (Alternatively, type `Circle[D,E,F]` in the Input field.

(8) Click and drag the vertices of your triangle to make sure the circle goes through *nine* (!) of the points you just constructed: the midpoints of the sides, the feet of the altitudes, and the midpoints between the vertices and the orthocenter. If your circle does not go through these points, look through your previous work for a mistake. Also make sure that the feet of your altitudes do not disappear when they are outside of the triangle, as in the second image below. If they do, go back and fix Step 4.
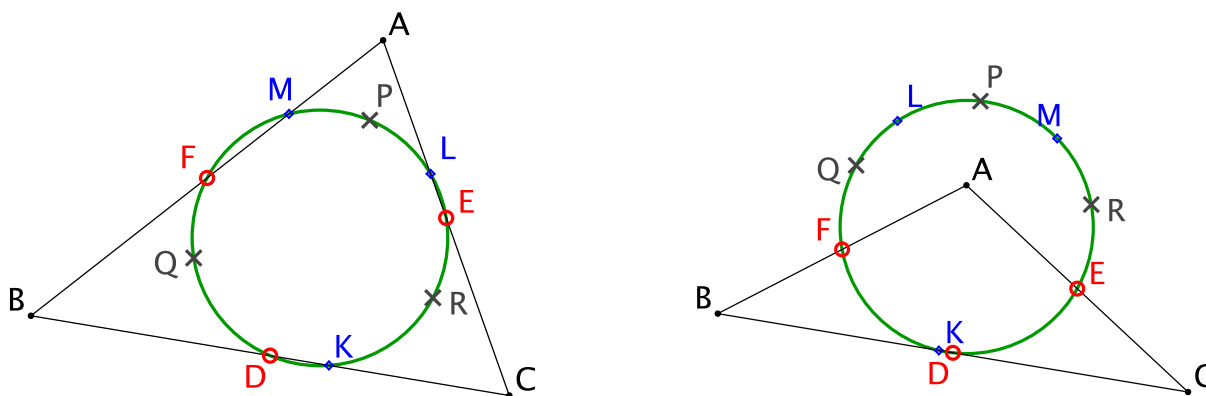
To receive credit for this assignment, save your file as `lastname-5335-lab4.ggb` and email it to me as an attachment by the beginning of class on Wednesday, 11/11/15. I'll use the following grading rubric out of 5 points:

- If I can: open your file; can see the nine-point circle; it updates correctly as I drag the vertices; you have named the points as above; but the feet of the altitudes disappear when they are out of the circle: 3 points.
- If, in addition, you have correctly defined the feet of the altitudes so they still appear even when they are outside of the triangle: 4 points.
- If, in addition, you have spent some time cleaning things up so that only the triangle, the circle, and the nine points are visible: 5 points (full credit). In other words, if you've left the altitudes, or the orthocenter, or unnecessary labels of line segments and circles, your score is capped at 4/5.
- If, an addition, you've spent a little time nicely styling your diagram, I'll consider a bonus point, for a total of 6. (In my images above, for example; the midpoints D, E and F are red, open circles; the feet K, L, and M are blue diamonds; the midpoints from the vertices to the orthocenter are ×'s. You don't need to match those choices; the point is to make it nice enough that you could include it in a report or a textbook.)