

This (fairly short) lab assignment is an interactive version of the example we did in class to investigate area in the Poincare Half Plane. We computed the Poincare area of the Euclidean unit square S given by $0 \leq x \leq 1$ and $n \leq y \leq n + 1$ for various values of n . The general formula turned out to be

$$\text{Poincare Area} = \iint_S \frac{1}{y^2} dA = \int_{-1}^1 \int_n^{n+1} \frac{1}{y^2} dy dx = \cdots = \frac{1}{n(n+1)}.$$

In this lab you'll plot a square in the Poincare Half Plane and drag it around to see the effect on its Poincare Area.

GEOGEBRA CONSTRUCTION

- (1) Open a new GeoGebra window. Use the View menu if necessary to turn **on** the axes and the grid. It will be useful to see both of them for this lab.
- (2) Create a point A away from the axes by using the *Point* tool, or typing $A=(1,1)$ in the input field.
- (3) Create a unit square which has A as its bottom-left corner. You could do this with the *Polygon* tool and careful clicking, or type

`Polygon[A, A+(1,0), A+(1,1), A+(0,1)]`

in the input field; that command tells GeoGebra to create a polygon whose four corners are A , $A + (1,0)$, and so on. Do you see why that would give you a unit square?

- (4) In our example, n was the minimum y -coordinate of our square. In our diagram, that's the y -coordinate of A . GeoGebra allows you to refer to the x and y coordinates of A with the expressions $x(A)$ and $y(A)$. So type

$$n = y(A)$$

in the input field.

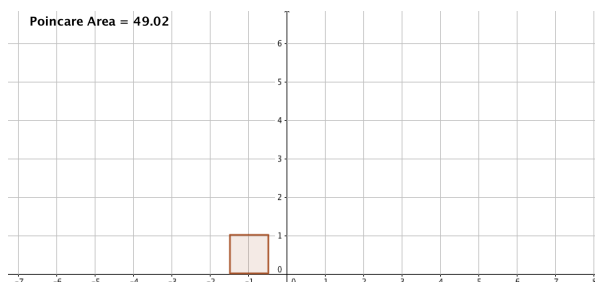
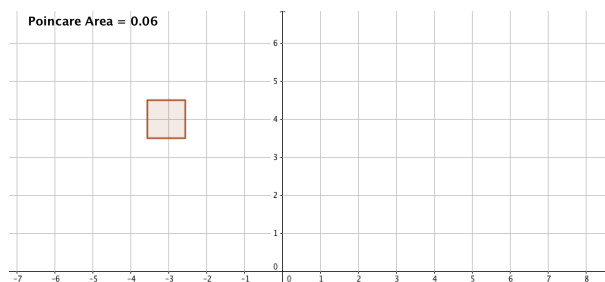
- (5) GeoGebra doesn't have a built-in tool to compute the Poincare area of a polygon, but we already found the general formula using calculus. Enter

$$\text{area} = 1 / (n(n+1))$$

in the input field. The result should be a number which is the Poincare area of your square.

- (6) Create text label in the upper left hand corner: **Poincare Area = (area)** where **(area)** is the area object you created in the previous step. (Reread the steps in Lab #5 if you need a reminder of how to insert variables into a text label.)
- (7) Click and drag your square around and make sure the area of the square updates. Experiment a bit. Because the formula for Poincare area doesn't depend on x , moving the square sideways shouldn't change its area. Sliding it up should make the area small; dragging it towards the x -axis should make it very large.

Here are two examples from my own GeoGebra file:



- (8) Make your diagram look nice. Notice the following features of mine, for example. (And you can probably improve on mine!)
- The Poincare Half Plane does not include points below the x -axis. This diagram shouldn't either; adjust your window size and/or drag the plane around to adjust.
 - The point A isn't relevant for the example. There's no need to display it.
 - Similarly, there's no need to display labels for the segments making up the square, or the square itself. Even worse, depending on your settings, GeoGebra might label them with numbers, which represent the *Euclidean* length of the segments and/or *Euclidean* area of the square. We're doing Poincare area here, so make sure those labels aren't there.

To receive credit for this assignment, save your file as `lastname-5335-lab6.ggb` and attach it to an email to me with subject line **Math 5335 Lab 6** by the beginning of class on Wednesday, 12/9/15. For full credit, I should be able to move your square and see that the label updates with its correct Poincare area. If your diagram has extra "cruff" in it, such as points, labels for segments, etc. but is otherwise ok, you'll receive 4 points. So spend a minute to make things look nice!