Updated 12/12/17: our methods for finding angles measures are in Chapter 3, not Chapter 2.

This is an open-book, -library, -internet take home exam. However, standard academic honesty rules apply. If you make use of an outside resource, keep the following in mind if you want to receive credit for your solution:

- Cite the resource as part of your solution. *Don't claim somebody else's work as your own.*
- You must still write the solution in your words. I'm interested in whether you understand the ideas, not whether you can transcribe somebody else's work. Here's a good guideline: wait 30 minutes; if you can write out your answer without using the outside resource, then you understand the ideas.
- Your solution must be consistent with the definitions and methods of this course. For example, Wikipedia has a page about the Poincare Half Plane, but the descriptions, notations and methods on that page are totally different than our book's. If you hand in a solution making use of Riemannian metrics, I can't count it for credit unless you also develop the subject of Reimannian metrics. So you're better off using our text book and notes from class.

Please take these guidelines seriously. I can use Google too.

You may not collaborate with others on the exam; I am the only person you are allowed to consult. You can ask questions during office hours, or you can email me at any time during the day.

Our final exam is scheduled for 1:30pm–3:30pm on Monday, December 18th. Instead of making the exam due on Monday, I'll have office hours at that time and I'll give you an extra day to do the exam. (I'd gladly give you more time, but I have my own deadline for when I need to grade finals and submit grades, and the clock starts ticking at our scheduled exam time on 12/18. If the deadline were any later, I wouldn't have time to finish.) In addition to my regular office hours on Tuesday, 12/12, I'll add office hours on Thursday and Friday, 12/14 and 12/15, from 1:30-3:00, and on Monday, 12/18 from 1:30-3:30pm.

Due: Tuesday, 12/19/2017 at 1:30pm in my mailbox in Vincent 107.

You should plan to spend at least as much time on this exam as you would have spent studying for (and taking) an in-class final. As always, you should explain your work, writing complete sentences with reasonably correct grammar. A good rule of thumb is that the work you hand in for this exam **should not be your first draft**. Figure out the problem on another sheet of paper, organize your thoughts, and then write out your solution.

You should answer questions in the spirit they are intended, using the appropriate methods. For example, when you're asked to find the measure of an angle using the definition or methods in Chapter 3, you should not argue that the angle happens to be the interior angle of a regular hexagon, and then cite Corollary 8.13 to say its measure is $4\pi/6 = 2\pi/3$. You can always ask me if you're not sure whether a method is appropriate or not! (It helps to start the exam early enough so you have time to ask questions.)

The exam covers Euclidean/Vector Geometry and Poincare Half Plane Geometry, with more of an emphasis on the latter since you've already been tested more on Euclidean/Vector Geometry.

Problems

Euclidean Geometry. In the following problems you should interpret all terms in the sense of the Euclidean vector geometry we developed up through Chapter 9. (40 Points Total)

Problem E.1: (15 Points) Use the definition or methods of Chapter 3 to find the measure of the angle $\angle(2,0)(0,0)(-\sqrt{2},-\sqrt{6})$. You may use the definition that $\arccos(-1) = \int_{-1}^{1} \frac{1}{\sqrt{1-t^2}}\,dt$ equals π, but you must develop any other angle measures that you use.

Problem E.2: (15 Points) Let $ABCD$ and $WXYZ$ be congruent rectangles in the Euclidean plane. Describe a sequence of 3 or fewer reflections which will be guaranteed to map $ABCD$ to $WXYZ$. Explain why your sequence works; illustrations will be helpful for full credit!

Problem E.3: (10 Points) A quadrilateral $ABCD$ is called *cyclic* if all four of its vertices are on a single circle in the Euclidean plane. Given three noncollinear points, describe the set of all points D such that $ABCD$ is cyclic and simple. Justify your answer.

Poincare Half Plane Geometry. The rest of this exam covers geometry of the Poincare Half Plane. For the remaining problems, you should interpret all terms (lines, angles, triangles, distances, isometries, etc.) in the context of Poincare Half Plane Geometry. (60 Points)

Problem P.1: (5 Points) Find four Poincare lines in the half plane, each of which is asymptotically parallel to both $(x-3)^2+y^2=4$ and $(x+1)^2+y^2=1$. Give the equations of your lines and show them in a sketch.

Problem P.2: (20 Points) Draw a careful, accurate picture of the triangle with vertices $(4,2)$, $(8,2)$ and $(8,\sqrt{20})$. Then find its area without directly computing an integral. Use exact values in your work, but at the end also compute your area to at least five decimal places. *(Hint: two angles in the triangle are very nice. The third isn't. Alas, we can't always have everything we want!)*

Problem P.3: (25 Points) Let $\mathcal{D}_{0,s}(x,y)=(sx,sy)$ be the function which dilates the Poincare Half Plane by scaling radially away from the origin by a factor of s. Let $\mathcal{T}_s(x,y)=(x+s,y)$ be the function which translates points in the Poincare Half Plane horizontally by s. Let $d(P,Q)$ denote the distance between two points P and Q in the Poincare Half Plane.

(a) Prove that $\mathcal{T}_s$ is an isometry of the Poincare Half Plane, i.e. prove that it preserves the Poincare distance between points. (7 Points)

(b) Prove that $\mathcal{D}_{0,s}$ is an isometry of the Poincare Half Plane. (10 Points)

(c) Prove that $\mathcal{D}_{\omega,s}$ is an isometry of the Poincare Half Plane, where $\mathcal{D}_{\omega,s}$ performs the same scaling, but from $(\omega,0)$ instead of $(0,0)$. (8 Points)

Problem P.4: (10 Points) Prove the triangle inequality holds in the Poincare Half Plane: given points P, Q and R,

$$d(P,Q)+d(Q,R)\geq d(P,R)$$

where $d(A,B)$ is the Poincare distance from A to B.

Hint: There are a lot of ways P, Q and R can be arranged with respect to each other. I wouldn't recommend approaching this problem by proving each of those individual cases. Instead, transform the general case into a more tractable situation with fewer things to check. This problem is intended to be more difficult than others; don't spend much time on it until you're certain that the rest of your exam is in good shape!