

In this lab exercise you will construct a demonstration of *Ceva's Theorem*, which is described in exercise 7.24. Because this is the fifth lab assignment, the directions below will include fewer details than previous labs. You can go back and re-read previous labs if you need some refreshers about how to adjust colors, add text labels whose values depend on values in your construction, and so on. Let me know if you have trouble figuring out which tool to use in any given step.

## GEOGEBRA CONSTRUCTION

- (1) Open a new GeoGebra window. Go to the View menu and **turn off the axes and the grid**, so the drawing pad is blank.
- (2) Create  $\triangle ABC$ .
- (3) Add a point  $P$  in the interior of your triangle.
- (4) Create rays  $\overrightarrow{AP}$ ,  $\overrightarrow{BP}$ , and  $\overrightarrow{CP}$ .
- (5) Use the *Intersect* tool to find the intersection of each ray with the opposite side of the triangle. Label the intersection points as follows:

$$X = \overrightarrow{AP} \cap \overline{BC}$$

$$Y = \overrightarrow{BP} \cap \overline{AC}$$

$$Z = \overrightarrow{CP} \cap \overline{AB}$$

- (6) Now the fun begins! In Ceva's Theorem we're interested in the product of three ratios:

$$\frac{|\overline{XB}|}{|\overline{XC}|} \cdot \frac{|\overline{YC}|}{|\overline{YA}|} \cdot \frac{|\overline{ZA}|}{|\overline{ZB}|}$$

Thus you need to define each of those segments in your diagram. You can do this with the *Segment* tool, clicking on  $X$  and  $A$  to create  $\overline{XA}$ , then  $A$  and  $C$ , and so on. If you prefer, you can use the input field to create segments via typed commands, like `Segment(A,X)`. GeoGebra automatically computes the length of each line segment, and you'll need these later. To make your life easier, name each of them according to your endpoints. For example,  $\overline{XB}$  should be named `xb`,  $\overline{XC}$  should be `xc`, and so on.

- (7) Ceva's Theorem tells us that

$$\frac{|\overline{XB}|}{|\overline{XC}|} \cdot \frac{|\overline{YC}|}{|\overline{YA}|} \cdot \frac{|\overline{ZA}|}{|\overline{ZB}|} = 1.$$

It's time to create a text label to show that this is true for your diagram! (Creating this label is the longest part of this lab; you might want to go re-read Lab #2 for a refresher about how to create text labels with values that change as you adjust your diagram.)

Use the *Text* tool to create a label, and paste the following text into the box. (GeoGebra should activate the "LaTeX Formula" button automatically, but you can check the box as needed.) The label is so long that I've had to include it on multiple lines:

$$\begin{aligned} & \frac{|\overline{XB}|}{|\overline{XC}|} \cdot \frac{|\overline{YC}|}{|\overline{YA}|} \cdot \frac{|\overline{ZA}|}{|\overline{ZB}|} \\ & = \frac{|\overline{XB}|}{|\overline{XC}|} \cdot \frac{|\overline{YC}|}{|\overline{YA}|} \cdot \frac{|\overline{ZA}|}{|\overline{ZB}|} \end{aligned}$$

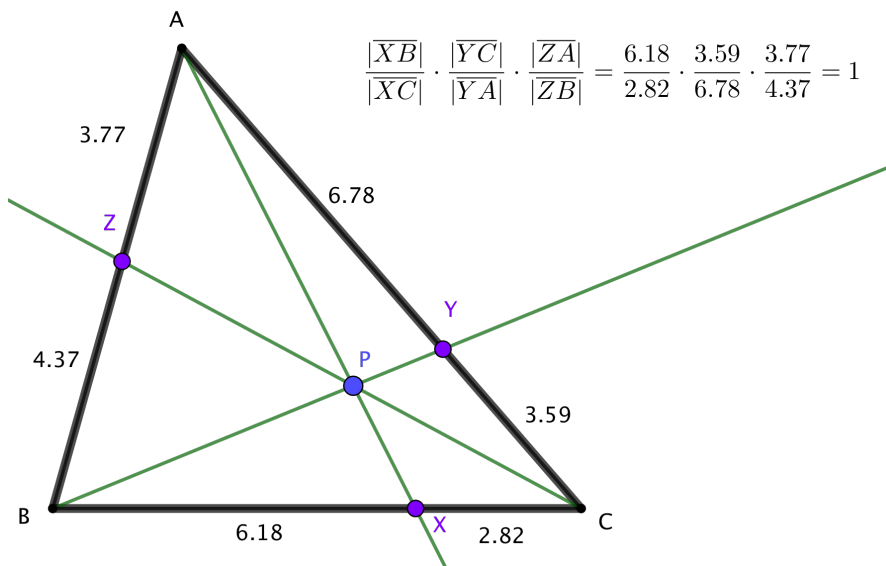
The first three lines are L<sup>A</sup>T<sub>E</sub>X code to typeset  $\frac{|XB|}{|XC|} \cdot \frac{|YC|}{|YA|} \cdot \frac{|ZA|}{|ZB|}$ . The bottom line includes the L<sup>A</sup>T<sub>E</sub>X code for blank fractions. It's up to you to fill them in!

In the first `\dfrac{ }{ }`, move your cursor to the first blank between the curly braces, and add the object `xb`, which represents the length of  $\overline{XB}$ . In the next blank space between curly braces, add the object `xc`. Now move on to the next `\dfrac{ }{ }` and add the objects `yc` and `ya`. In the third `\dfrac{ }{ }` you'll add the objects for `za` and `zb`.

If you're curious, but haven't learned L<sup>A</sup>T<sub>E</sub>X (which is fine!): `\frac{a}{b}` is the code for a fraction  $\frac{a}{b}$ . The code `\dfrac{a}{b}` gives the same fraction in *display* mode, which is a bit larger:  $\frac{a}{b}$ .

At this point you should not only see  $\frac{|XB|}{|XC|} \cdot \frac{|YC|}{|YA|} \cdot \frac{|ZA|}{|ZB|}$ , but also three fractions with the lengths of each line segment copied in. You're almost done! Now go to the end, after the last `=`, and add an object. (Any object – it doesn't matter which.) Click inside that object, and enter the formula `xb * yc * za / ( xc * ya * zb)`.

- (8) If your labels are displaying correctly, and the formula gives a result of 1, including when you move the vertices or  $P$ , congratulations! You've now demonstrated Ceva's Theorem.<sup>1</sup> If the fractions aren't correct, or your product is not equal to one, find the error and adjust as necessary. Here's an example of my construction, for comparison:



- (9) Go back and clean up your diagram so that it would be appropriate to use in a presentation to a class; you don't need to have the names of various rays and line segments, for example, although it's useful to display the values of the six line segments. You can use my construction above as a guide, but feel free to improve on it as necessary.

To receive credit for this assignment, save your file as `lastname-5335-1ab5.ggb` and email it to me as an attachment by the beginning of class on Wednesday, 11/15/17. I'll use the following grading rubric out of 5 points:

- 2 Points:** I can open your file, see the triangle and the points, and they update as I drag  $P$  or the vertices, but the formula in the text label isn't correct. (Or, the `= 1` at the end is just text, instead of an actual computation!)
- 3 Points:** I can open your file, and the construction is correct, including the text label, but the diagram is confusing because you've left labels of various segments, aren't displaying the lengths of the six smaller line segments, etc.
- 5 Points:** I can open your file, the construction is correct, and it's been cleaned up appropriately.

<sup>1</sup>Here's what Ceva's Theorem actually says: if you connect  $A$ ,  $B$  and  $C$  to points  $X$ ,  $Y$  and  $Z$  on the opposite sides, those new line segments (called *cevians*) are concurrent at a point if and only if the given product of ratios is 1. In our construction, we *forced* the segments to be concurrent at point  $P$ , so the product should always be 1.