In this lab you're going to create a simpler, less-capable version of this GeoGebra demonstration:

https://www.geogebra.org/classic/RvuHbc8P

You should start by clicking on that link and experimenting with the hyperbolic triangle there. Drag vertex B to the right until it's directly above C. Notice how the Poincaré line segment seamlessly changes between a piece of a circular arc and a portion of a vertical ray. Drag all of the vertices around to see the different shapes that qualify as a "triangle" in the Poincaré Half Plane. Pay careful attention to the sum of the angles. Can you make the angles add to less than 5°? Can you make them add up to nearly 180°? *(Hints: try putting all three vertices near the x-axis, and far apart. Next, try placing all three vertices close together, far away from the x-axis – up near $y = 10$, for example.)*

Make sure you can answer the following questions, which will be relevant on Exam 2:

- Why does a triangle near $y = 10$ have a smaller Poincaré area than a similar looking triangle near $y = 1$?
- Given the fact that $\|\triangle ABC\| = \pi - \alpha - \beta - \gamma$, why did you have to draw a very large triangle to make the angle sum less than 5° and a small triangle to make it close to 180°?

GeoGebra does not include any built-in tools for Poincaré Half Plane lines, triangles or angles. The person who created that demonstration spent a significant amount of time programming new Hyperbolic Geometry Tools for GeoGebra, which is beyond the scope of Math 5335. This lab will allow you to create your own hyperbolic triangle with draggable vertices, but it will not be as robust. It will only work if the vertices are in a specific order, and it won't allow for the possibility of vertical line segments. If you have a programming background, feel free to expand on this and create a more comprehensive demonstration!

GeoGebra Construction

(1) Open a new GeoGebra window. Use the View menu if necessary to turn **on** the axes and the grid. It will be useful to see both of them for this lab. Because we're working with the Poincare Half Plane, you can drag the grid until the x-axis is near the bottom of your window, so that you're only looking at points with positive y values.

(2) Create the following points, either by using the `Point` tool, or typing (e.g. "`A=(-2,2)`") in the input field:

$$A = (-2, 2)$$
$$B = (2, 2)$$
$$C = (5, 4)$$

(3) The Poincaré line $\overleftrightarrow{AB}$ will have the form $(x - \omega)^2 + y^2 = \rho^2$. Back in Chapter 10 (page 171), we found formulas for ω and ρ so that the line $(x - \omega)^2 + y^2 = \rho^2$ will pass through points (a, b) and (c, d). In particular:

$$\omega = \frac{(d^2 - b^2) + (c^2 - a^2)}{2(c - a)} = \frac{1}{2} \cdot \frac{c^2 + d^2 - a^2 - b^2}{c - a}$$

Let's create a function in GeoGebra so that the computer can do this computation for us. Copy and paste the following into the input field, or else type it exactly as it appears here:

```
omega(a,b,c,d)=(1/2)(c^2+d^2-a^2-b^2)/(c-a)
```

(4) Let's create $\overleftrightarrow{AB}$. Recall from an earlier lab that you can write `x(A)` and `y(A)` for the coordinates of a point A. So the expression `omega(x(A),y(A),x(B),y(B))` will give ω for the line $\overleftrightarrow{AB}$. Type

```
( omega(x(A),y(A),x(B),y(B)), 0 )
```

in the input field. This will create a new point, likely called D, which is the center of $\overleftrightarrow{AB}$. Notice that we never computed ρ. That's because GeoGebra can do it for us. Use the `Circle with Center through Point` tool to create aa circle centered at D, through A. It will automatically include B as well. (Alternatively, type `Circle(D, A)` in the input field to create the circle.)

(5) Do similar work to create lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$.

(6) Drag A, B and C around to double check that the circle centers and circles all update correctly. If you look carefully, you can see the hyperbolic triangle $\triangle ABC$ in the midst of all the circles, but it's a bit messy.

(7) Now let's fix the mess, and only displace Poincaré line segments instead of the entire lines. Return A, B and C to their original locations, and type the following commands in the input field:

```
CircularArc(D, B, A)
CircularArc(E, C, A)
CircularArc(F, C, B)
```

Here I'm assuming D, E and F are the centers of the Poincaré lines $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, and $\overleftrightarrow{BC}$, in that order. If your centers have different names, update the commands appropriately. Make sure not to change the order of any of the points, or your diagram will not be correct.

(8) Hide the circles you created earlier. Select the circular arcs – likely named f, g and h – and make them green. Then adjust their style so that they are as thick as possible.

(9) At this point you can drag A, B and C around and marvel at the beauty of your hyperbolic triangle. It will look great as long as A is somewhere to the left of B, and B is somewhere to the left of C. Try moving B until it's directly above C. You'll notice the circular arc disappears, and it's *not* replaced with a vertical line segment. You'd have to do more programming for that! Now move B further to the right, and things go *totally* crazy. Unhide the circles and you'll see what's happening; GeoGebra gets confused, and shows you the wrong part of the circle!

(10) Make your diagram look nice. Move A, B, and C back to their original locations. Hide the circles, and the circle centers (D, E and F). Make sure you've adjusted the window size and/or dragged the plane around so that you're only looking at the top half of the plane.

To receive credit for this assignment, save your file as `lastname-5335-lab6.ggb` and attach it to an email to me with subject line `Math 5335 Lab 6` by the beginning of class on Wednesday, 12/5/18. For full credit, I should be able to move your vertices and see that the hyperbolic triangle updates appropriately. If your diagram has extra "cruft" in it, such as circles or their centers, but is otherwise ok, you'll receive 4 points. So spend a minute to make things look nice!