The following is a non-comprehensive list of solutions to homework problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

1.5.4: If k and l are perpendicular, they must have direction vectors U and V which are perpendicular – i.e. $\langle U, V\rangle = 0$. Let m be any line which is parallel to l. By definition, a direction vector W of m must be a multiple of V: $W = cV$, for some c. But then $\langle U, W\rangle = \langle U, cV\rangle = c\langle U, V\rangle = 0$, so k is perpendicular to m.

1.5.6: The given line goes through the point $(15, -8)$, and has a direction vector $(-7, -17)$. Hence the vector $(17, -7)$ is perpendicular to the line, and the normal form is:

$$\langle (17, -7), X - (15, -8)\rangle = 0$$

Other answers are possible; $(17, -7)$ may be replaced with any nonzero multiple of itself, you can distribute the dot product and have $\langle (17, -7), X\rangle = 311$, etc.

1.5.7: This problem is essentially the reverse of 1.5.6. From the given information, the line has a direction indicator U which is perpendicular to $(3, -14)$, so $U = (14, 3)$ would work. Finding a point on the line is a bit trickier. One possible approach is to guess that there might be a point on the line for which $X = (x_1, 0)$, in which case

$$\langle (4, -13), X\rangle = \langle (4, -13), (x_1, 0)\rangle = 4x_1 = 5$$

which means $x_1 = 5/4$ and imples that $X = (5/4, 0)$ is on the line. Hence a parametric form of the line would be $(5/4, 0) + s(14, 3)$.

1.5.9: As discussed in class, the important thing here was to actually cite a proposition and calculations to justify your work; it's not enough to draw a picture and say, "Well, obviously this must be true." The easiest way (in my mind) is to use Proposition 19. So to prove that "being on the same side" of some given line $l : \{\langle A, X - Y\rangle = 0\}$ is an equivalence relation:

- $\langle A, P - Y\rangle$ and $\langle A, P - Y\rangle$ certainly have the same sign. (They're equal!)
- If $\langle A, P - Y\rangle$ has the same sign as $\langle A, Q - Y\rangle$ then $\langle A, Q - Y\rangle$ has the same sign as $\langle A, P - Y\rangle$.
- If $\langle A, P - Y\rangle$ has the same sign as $\langle A, Q - Y\rangle$ and $\langle A, Q - Y\rangle$ has the same sign as $\langle A, R - Y\rangle$, then $\langle A, P - Y\rangle$ has the same sign as $\langle A, R - Y\rangle$.

Conversely, "being on opposite sides" of l fails to be an equiavlence relation immediately, because a point P will not be related to itself: $\langle A, P - Y\rangle$ does not have a different sign than $\langle A, P - Y\rangle$. (Duh... but that needs to be written out!)

1.5.17: Proving Proposition 31 from Lemma 30 is almost as easy as defining $U = Q - P$, $V = R - Q$ and figuring out which line segments correspond to which vectors. You also need to explain why "Q is between P and R" is equivalent to "one of U and V is a nonnegative scalar multple of the other" to prove the last statement about equality.

1.5.28: Suppose $l = \{Q + sU\}$. We know the line $k = \{P + sU\}$ is parallel to l (since it has the same direction indicator) and is incident with P. It remains to prove that this is the *unique* such line. Let m be any line which through P which is parallel to l. It must have a direction indicator which is a non-zero multiple of U, $V = cU$. You now need to make any argument explaining why m is in fact the same line as k. (The third part of Proposition 3 can be used here.)

1.5.42: Here is one possible approach, which avoids any calculations with the components of U and V. First, assume $\langle U, V\rangle = 1$. To prove $U = V$ it suffices to show $||U - V|| = 0$ or, equivalently, $||U - V||^2 = 0$. Using Lemma 26,

$$||U - V||^2 = ||U||^2 + ||V||^2 - 2\langle U, V\rangle = 1 + 1 - 2(1) = 0$$

To prove the second assertion about $U = -V$ you would use the alternate equation in Lemma 26.