This is an open-book, -library, -internet take home exam. (However, standard academic honesty rules apply. If you use a source to help you solve a problem, cite it in your solution. *Dont claim somebody else's work as your own.*) You are not allowed to collaborate; I am the only person you are allowed to consult. You can ask questions during office hours, or you can email me at any time during the day.

You should plan to spend at least as much time on this exam as you would have spent studying for an in-class final; because you will have had the exam for a full week (and get to hand it in three days after the regularly scheduled time for an in-class final exam) you should complete each problem. As always, you should explain your work, writing complete sentences with reasonably correct grammar. A good rule of thumb is that the work you hand in for this exam **should not be your first draft**. Figure out the problem on another sheet of paper, organize your thoughts, and then write out your solution.

Due: Tuesday, 12/21/2010 at 1:30pm in my mailbox in Vincent 107.

Problems

Note: because our previous midterms focused on Euclidean geometry, this final covers the geometry of the Poincare Half Plane. Hence all terms below (triangle, line, area, etc.) should be interpreted in the context of Poincare Half Plane Geometry, not the Euclidean Geometry of Chapters 1-8.

10.5.7: (14 Points)

10.5.29: (16 Points)

10.5.38: (16 Points) (See online hint)

Problem 1: (24 Points) Draw a careful, accurate picture of the triangle with vertices $(3, 2\sqrt{6})$, $(8, 2\sqrt{6})$ and $(8, 7)$. Then find its area without directly computing an integral. Use exact values in your work, but at the end also compute your area to at least five decimal places.

Problem 2: (14 Points) In class we found the area of a hyperbolic regular pentagon whose interior angles were all $\pi/2$; this pentagon could tessellate the Poincare Half Plane by arranging four of them around each vertex. It is a fact that, given an **even** integer $k > 2$, there exists a hyperbolic regular pentagon which can tessellate the Poincare Half Plane by arranging k of them around each vertex. Find the area of such a polygon in terms of k.

(Hint: our example in class was the case where $k = 4$. For this problem, you may assume without proof that a regular pentagon can be decomposed into triangles by drawing diagonals from one vertex.)

Problem 3: (16 Points) Let $\mathcal{D}_{0,s}(x, y) = (sx, sy)$ be the function which dilates the Poincare Half Plane by scaling radially away from the origin by a factor of s. (8 Points each)

(a) Prove that $\mathcal{D}_{0,s}$ is an isometry of the Poincare Half Plane, i.e. prove that it preserves the Poincare distance between points.

(b) Prove that $\mathcal{D}_{\omega,s}$ is an isometry of the Poincare Half Plane, where $\mathcal{D}_{\omega,s}$ performs the same scaling, but from $(\omega, 0)$ instead of $(0, 0)$.