The following is a non-comprehensive list of solutions to homework problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

8.4.3: You can almost use the formula given in the book for a circle inversion, with one exception: that formula assumes the mir ror is centered at the origin, whereas our circle is centered at $(-8, 13)$. So we need to first move everything so the center is at th e origin, then invert, and then move it back:

$$\mathcal{I}(X) = \begin{cases} \frac{\rho^2}{||X-C||^2}(X - C) + C, & X \neq C, \infty \\ \infty & X = C \\ C & X = \infty \end{cases}$$

where $C = (-8, 13)$ and $\rho = 29$ in our case. By my quick calculations, this yields:

$$\mathcal{I}(0,0) = \left(\frac{4864}{233}, \frac{-7904}{233}\right)$$

$\mathcal{I}(12, -8) = (12, -8)$ (this point is on the mirror, so it stays fixed!)

$$\mathcal{I}(\infty) = (-8, 13)$$

$$\mathcal{I}(8, -13) = \left(\frac{1500}{233}, \frac{-4875}{233}\right)$$

8.3.4: I've listed the answers below; ask me if you have trouble finding these or drawing the accompanying pictures.

(i) ($||X|| = 2$) This circle does not go through the origin and is sent to another circle which does not go through the origin, specifically $||X|| = 1/2$.

(ii) ($||X|| = 3$) Similarly, this circle is sent to the circle $||X|| = 1/3$.

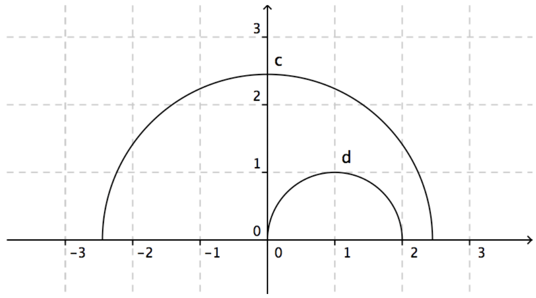
(iii) ($||X - (0, -1)|| = 1$) This circle does go through the origin, so it is sent to a line which does not go through the origin, specifically $y = -1/2$.

(iv) ($y = -1$) This line not through the origin goes to a circle which does go through the origin, specifically $||X - (0, -1/2)|| = 1/2$.

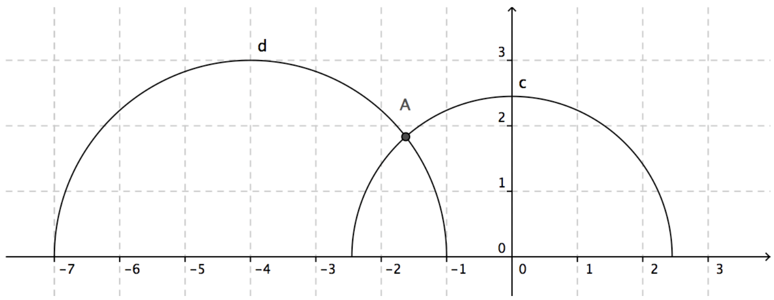
(v) ($y = x$) This line through the origin is sent to itself, $y = x$.

(vi) ($||X - (-3, 0)|| = 5/2$) This circle does not contain the origin and is sent to another such circle, $||X - (-12/11, 0)|| = 10/11$.

9.9.7: These two lines do not intersect, as is evident from the picture below. However, for full credit you needed to show algebraically that no point (x, y) satisfies both equations. Ask me if you're not sure how to do this.

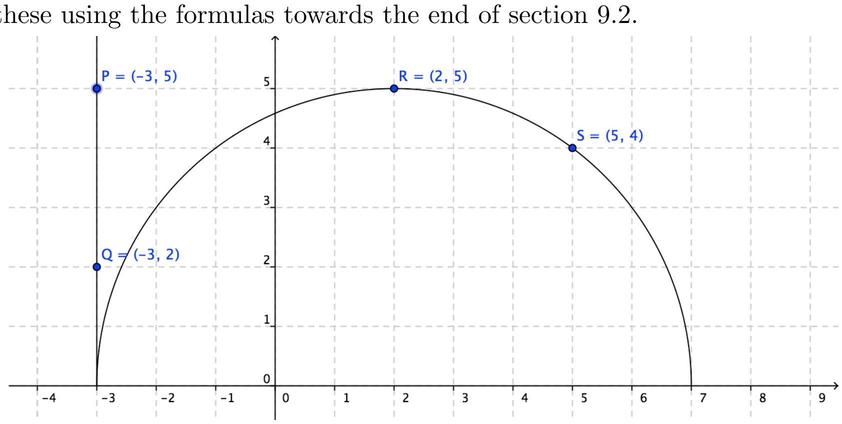


9.9.8: These two lines *do* intersect, as you can see below. Solving the algebraic system of equations gives the point $\left(-\frac{13}{8}, \frac{\sqrt{215}}{8}\right)$. Again, ask me if you have trouble with the calculations.



9.9.14: People chose many ways to rephrase the stated proposition; the main idea is that lines are sets of points, and if two lines ℓ and k "meet" at a point P, then $P \in \ell$ and $P \in k$. Now suppose ℓ and k both contain P and Q. By I.1, there is one unique line which contains both of them. Hence $\ell = k$. So any two lines which meet at two (or more) points must be equal; stated differently, distinct lines can meet at most one point.

9.9.22: The first two points are on the line $x = -3$, while the second two are on the line $(x-2)^2+y^2 = 25$. You can find these using the formulas towards the end of section 9.2.



These lines are asymptotically parallel, to use the language we developed in Chapter 11; they *would* intersect at $(-3, 0)$, except $y > 0$ on both lines, so there is no actual point of intersection.