In this lab you will investigate one of the characterizations of the interior of an angle in Theorem 1.38. You should read it on pp29-30 of your textbook before continuing. As with Lab 1, this activity shouldn't take too long. Make sure to follow directions below carefully. Most of the issues in Lab 1 arose when people skimmed through the text too quickly and left out a step or two. You'll continue to learn about new GeoGebra commands (like `Vector`) and objects (such as *sliders*), but I've spelled things out carefully below so you don't have to spend time online reading the GeoGebra manual.

GeoGebra Construction

(1) Open a new GeoGebra window. Go to the View menu and turn the Grid on and the Axes off.

(2) Choose the Point tool and create three noncollinear points. Spread them out a bit so you have room to create other objects. GeoGebra will name them A, B and C, but to be more consistent with our book's notation, go back and rename them P, Q, R. [Remember, you can rename objects by right-clicking on them in the drawing window or in the Algebra panel on the left; you can also go to the Properties Editor in the Edit menu, choose the object, and give it a new name. With points there is also a shortcut: when you click and create a point (say, A), if the very next thing you do is type another letter (say P), GeoGebra will give you a popup window to let you change the name immediately.]

(3) Create $\angle PQR$ by drawing the appropriate rays. Either type `Ray[Q,P]` and `Ray[Q,R]` in the Input field or use the Ray tool and the mouse. Rename the rays p and r, respectively.

(4) We want to find direction indicators for each ray. You can create a vector which starts at Q and ends at P in one of two ways: either type `Vector[Q,P]` in the Input field, or choose the Vector tool (in the drop down menu below the line through two points) and click on Q, then P. Try one of these methods. **However**, we often want to have *unit* direction vectors, so we might as well do that now. Click the Undo button (or choose Undo from the Edit menu) to go back to the end of Step 3.

GeoGebra has a `UnitVector[]` command which takes a given vector and returns a unit vector in the same direction with the same starting point. We can create our unit direction indicators with the following commands in the Input field. (Note that we're telling GeoGebra to turn the vector $(Q-P)$ into a unit vector without actually creating/drawing $(Q-P)$ on the screen. Tricky!)

```
U = UnitVector[ Vector[Q,P] ]
W = UnitVector[ Vector[Q,R] ]
```

(5) We want the direction indicators to stand out from the rest of the drawing. Go to the Properties editor and choose the vectors. Under "Style" make them thicker; try 5 pixels. Under "Color" make them red.

(6) Part (iv) of Theorem 38 says that Y is in the interior of $\angle PQR$ if and only if it can be written in the form $Y = Q + aU + cW$ for $a, c > 0$. Hence we need a way to create variables a and c which can have different values. GeoGebra can do this with *sliders*. Choose the Slider tool (top square in the second drop-down menu from the right – it shows a line with a point and a label a). Click in the corner of your drawing and a window will appear. Name your slider a and choose minimum and maximum values of 0 and 10, respectively. Now create a second slider named c immediately below the first one, using the same minimum and maximum values.

(7) Now enter `Y = Q + a*U + c*W` in the Input field. You should see a point Y in the interior of the angle. Click and drag the sliders to change the values of a and c. Notice how Y moves when you do that, and see if you can understand why Y will always remain in the interior of the angle – i.e. the "smaller of the two possible angles formed by p and r. Move P and Q to see how your construction changes for acute and obtuse angles. What if your rays are opposite, or if you try to make $\angle PQR$ a reflex angle?

To receive credit for this assignment, save your construction as `lastname-5335-lab2.ggb` and email it as an attachment to <rogness@math.umn.edu>. **The subject line should be the same as the filename**, or my email filter might miss your message. The email is due by midnight on Friday, 10/15/10.