Due: Wednesday, 10/2/19 at the *beginning* of class.

In general, answers to homework problems should include any computations necessary to get the final answer and an explanation of your work. Some problems may be entirely computational, with very little writing, whereas others are proofs or explanations with little computation. If you're in doubt about what's required for a particular problem, ask me. As a rule of thumb, common sense prevails. This is a 5000 level course, not college algebra, so I don't need to see your work in excruciating detail. If you need to solve a system of equations as part of a problem, just tell me what the solution is; don't include a page of work showing every step of the Gauss-Jordan reduction of an augmented matrix.

When you write explanations, you should write in complete sentences with (reasonably) correct grammar. Granted, this is not a writing intensive course, but it is a 5000-level mathematics course, and at this level you're expected to be able to explain your work in a coherent, organized and logical manner.

Starred exercises in the textbook have answers in the back, ranging from quick hints to full solutions. If I assign any of those, explaining your reasoning becomes even more important; you should enhance, and not just transcribe, the solution in the back. In other cases it might be a good idea to do those problems and check your answers before working on the assigned problems, as a way to check your understanding.

In this course, vectors and points are always two-dimensional unless otherwise specified.

Chapter 3: Problems 3.8, [3.15 and 3.16] (will be considered as problem when grading), 3.22

Chapter 4: Problems 4.1, 4.10

A: (Not in your book): Let $\mathcal{U}$ and $\mathcal{V}$ be isometries on $\mathbb{R}^2$. Prove that the composition $\mathcal{U} \circ \mathcal{V}$ is an isometry.

B: (Not in your book): Let $\mathcal{U}$ be an isometry on $\mathbb{R}^2$. Prove: if $a + b + c = 1$, then

$$\mathcal{U}(aP + bQ + cR) = a\mathcal{U}(P) + b\mathcal{U}(Q) + c\mathcal{U}(R)$$

for any points $P, Q, R \in \mathbb{R}^2$.

Hints: This is the "three-term" version of Lemma 4.4, and you can cite Lemma 4.4 in your solution. A useful first step, assuming $a + b \neq 0$, could be to rewrite $aP + bQ$ as

$$aP + bQ = (a + b)\left[\frac{a}{a+b}P + \frac{b}{a+b}Q\right].$$

Put differently, this means $aP + bQ + cR = (a + b)S + cR$, for $S = \dfrac{a}{a+b}P + \dfrac{b}{a+b}Q$.

At some point in your proof, you should expect to apply Lemma 4.4.

Don't forget to deal with the case where $a + b = 0$! You shouldn't have to re-do your entire proof in this case, but should explain how you'd deal with it.