Due: Wednesday, 10/31/18 at the *beginning* of class.

In general, answers to homework problems should include any computations necessary to get the final answer and an explanation of your work. Some problems may be entirely computational, with very little writing, whereas others are proofs or explanations with little computation. If you're in doubt about what's required for a particular problem, ask me. As a rule of thumb, common sense prevails. This is a 5000 level course, not college algebra, so I don't need to see your work in excruciating detail. If you need to solve a system of equations as part of a problem, just tell me what the solution is; don't include a page of work showing every step of the Gauss-Jordan reduction of an augmented matrix.

When you write explanations, you should write in complete sentences with (reasonably) correct grammar. Granted, this is not a writing intensive course, but it is a 5000-level mathematics course, and at this level you're expected to be able to explain your work in a coherent, organized and logical manner.

Starred exercises in the textbook have answers in the back, ranging from quick hints to full solutions. If I assign any of those, explaining your reasoning becomes even more important; you should enhance, and not just transcribe, the solution in the back. In other cases it might be a good idea to do those problems and check your answers before working on the assigned problems, as a way to check your understanding.

In this course, vectors and points are always two-dimensional unless otherwise specified.

A: Suppose vector U is perpendicular to line ℓ. Prove $\mathcal{M}_\ell \circ \mathcal{T}_U$ is a reflection across a line k, where k is the line ℓ translated by $-U/2$, i.e. $k = \mathcal{T}_{-U/2}(\ell)$.

Chapter 7: Problems 7.4, 7.6. 7.13

B: Prove that the medians of a triangle subdivide the triangle into six smaller triangles of equal area, via the following steps. (This is Problem 7.26, with some scaffolding added in.) Given $\triangle ABC$, let $D = (A+B)/2$, $E = (B+C)/2$ and $F = (A+C)/2$ be the midpoints of its sides, and G be its centroid. Recall that $\|\triangle ABC\|$ denotes the area of the triangle.

(1) Prove $\|\triangle ADC\| = \|\triangle BDC\|$. This is an example of the *Same Base, Same Height Theorem*, i.e. two triangles whose bases have the same length, and which have the same height, have equal area. You can use this theorem in the rest of this problem for *any* two triangles with the same base and height, not just $\triangle ADC$ and $\triangle BDC$.

(2) Prove $\|\triangle ADG\| = \|\triangle BDG\|$ and, similarly, $\|\triangle BEG\| = \|\triangle CEG\|$ and $\|\triangle AFG\| = \|\triangle CFG\|$.

(over)

(3) Prove that all six triangles in the previous step have the same area. To avoid cumbersome notation, you might wish to denote each area with a letter, e.g.

$$x = \|\triangle ADG\| = \|\triangle BDG\|$$
$$y = \|\triangle BEG\| = \|\triangle CEG\|$$
$$z = \|\triangle AFG\| = \|\triangle CFG\|$$

Now use your diagram and the previous parts to show $y = z$, $x = z$, and $x = y$–in other words, $x = y = z$.

Hints / Reminders. Make sure to do the entire problem. Problem 7.6 is an "if and only if" statement, for example, so you have to write proofs in both directions. But Problem 7.4 also requires two directions. Once you've figured out your answer you need to explain why that kind of triangle has its orthocenter at a vertex – but then you need to explain why no *other* kind of triangle does. Put differently, once you figure out what goes in the blank, you're trying to prove the statment *the orthocenter of $\triangle ABC$ is at a vertex if and only if $\triangle ABC$ is* ________ .

For 7.13, you should show more work than a GeoGebra picture for your answer, but you don't have to write out systems of equations and their solutions. Once you demonstrate that you know the equations of perpendicular bisectors, altitudes, or whatever, then you can use a calculator, Wolfram Alpha, GeoGebra, or any other tool to find their intersection. (Make your life easy by choosing convenient forms of the equations of the lines. For example, for a median, you know a vertex and can quickly compute the midpoint on the opposite side. Now you have two points, so you can find a direction indicator and write down the parametric/vector equation of the line. There's no need to continue further and find [for example] the y-intercept equation for the line.) You can read about the Euler Line on the top of page 127.

For [A], class notes should be very helpful. As part of our classification of isometries we proved that, for $U \perp \ell$, $\mathcal{T}_U \circ \mathcal{M}_\ell = \mathcal{M}_k$, where $k = \mathcal{T}_{U/2}(\ell)$. The order of $\mathcal{T}_U$ and $\mathcal{M}_\ell$ are switched here, which is why $U/2$ changes to $-U/2$, but the proof will be very similar!

For [B], a well drawn and labeled diagram will be very helpful when figuring out the solution, and I'd consider it obligatory in any full-credit solution, if you expect a reader to be able to follow your work.