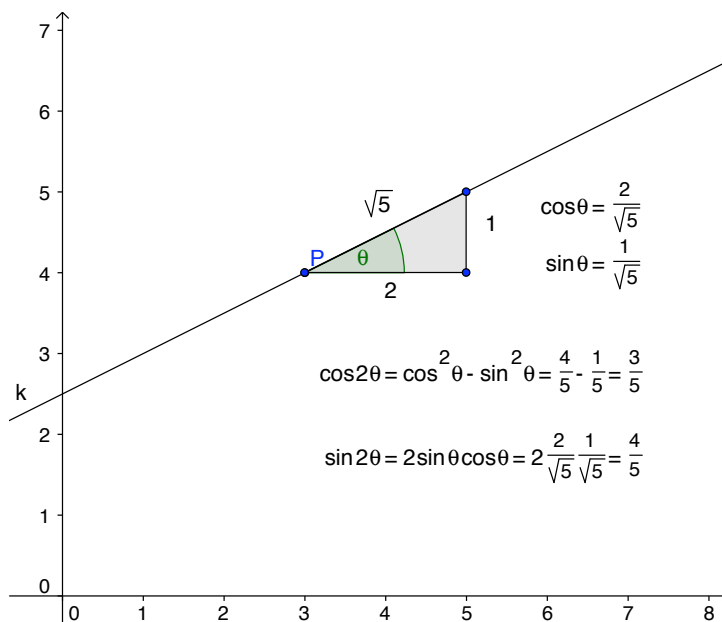In class we constructed the matrix formula for the reflection across the line $k = \{(3, 4) + t(2, 1)\}$ – i.e. the line through $P = (3, 4)$ with direction indicator $U = (2, 1)$. The picture below shows both the line and how to find $\cos \theta$, $\sin \theta$, $\cos 2\theta$ and $\sin 2\theta$, where θ is the angle formed by k and the horizontal.

Using the formula defined in class (together with the trig values found above),

$$\mathcal{M}_k(X) = F_\theta(X - P) + P$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \left(X - \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right) + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \left(X - \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right) + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

If we multiply this out, we have:

$$\mathcal{M}_k(X) = \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} X - \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} X - \begin{bmatrix} 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} X + \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

We can test our formula with two calculations:

$$\mathcal{M}_k \left(\begin{bmatrix} 5 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \cdots = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$$

$$\mathcal{M}_k \left(\begin{bmatrix} 0 \\ 5 \end{bmatrix} \right) = \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} + \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \cdots = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

You can check on the picture above that the reflection of $(5,0)$ across k is in fact $(1,8)$, and the reflection of $(0,5)$ is $(2,1)$. One further note: occasionally we might write the vector X in terms of its components x and y, in which case the formula takes the form:

$$
\mathcal{M}_k\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \cdots = \begin{bmatrix} \frac{3}{5}x + \frac{4}{5}y - 2 \\ \frac{4}{5}x - \frac{3}{5}y + 4 \end{bmatrix}
$$

Or, writing our vectors out with parentheses instead:

$$
\mathcal{M}_k(x,y) = \left(\frac{3}{5}x + \frac{4}{5}y - 2, \frac{4}{5}x - \frac{3}{5}y + 4\right)
$$