Implicit in any homework problem is that you must explain why your answer is correct, even if the problem does not ask for a formal proof. You should write in complete sentences with reasonably correct grammar. Math 5345 is not a writing intensive course, but it *is* a 5000-level mathematics course, and at this level you're expected to be able to explain your work in a coherent, organized and logical manner.

One short note: as you read the problems and the book, you might notice the word "metrizable." I touched on this very briefly in class, but it's worth mentioning here as well. Given a metric d on a space X we can always come up with a standard metric topology, where the open sets are unions of open balls $\{x \mid d(x, x_0) < r\}$, and so on. Given any general topological space X with a topology (i.e. list of open sets) τ, we're sometimes interested in whether or not τ would be the standard metric topology for some (unknown) metric d on X. If so, the topological space (X, τ) is *metrizable*; otherwise, it is not.

Homework Assignment

Section 7.2: 4, 10, 11, 12

Section 7.5: 2

A: We constructed the Sorgenfrey topology on $\mathbb{R}$ using $\mathcal{B} = \{[a, b) \mid a < b, a, b \in \mathbb{R}\}$. Show that $\mathcal{B}$ is, in fact, a basis for a topology on $\mathbb{R}$.

K1: Let $A \subset X$ be a closed set of a topological space X. Let $B \subset A$. Prove that B is closed in A (using the relative topology) if and only if B is closed in X.

K2: Let $X = \{y \geq 0\} \cup \{(0, -1)\} \subset \mathbb{R}^2$. Show that there is a ball $B_r(x_0, y_0)$ whose closure in X is strictly smaller than

$$\{(x, y) \mid d\left((x_0, y_0), (x, y)\right) \leq r\}$$

(So in metric spaces we need to be careful: the closure of an open disk might not be equal to the closed disk of the same radius.)

Jonathan Rogness <rogness@math.umn.edu> October 26, 2007