Your second exam is on Friday, November 16th. Officially it covers the following sections of your book: 7.2, 7.5, 3.1, and 3.2. (Sections 3.3 and 3.4 will not be on this exam.) It also includes the supplemental material I added about general topological spaces, including broader and deeper coverage of open sets, closed sets, closures, interiors, exteriors, bases, subbases, and so on. Because (1) it's harder to study supplemental material not in the main textbook and (2) this exam snuck up on us, I'm willing to give a bit more guidance than normal about the exam.

There may be some short answer or true/false questions based on definitions or simple facts. Hence you should know and be able to interpret the important definitions and characterizations of terms such as surface (or 2-manifold), topology, basis, subbasis, open, closed, closure, interior, exterior, continuous, homeomorphism, etc.

The long answer problems will be a subset of the following problems (or minor variations thereof):

1. Show that the connected sum $T^2 \# T^2$ of two tori can be represented by a octagonal disk with sides identified according to the word $aba^{-1}b^{-1}cdc^{-1}d^{-1}$.
2. (a) Show that the connected sum $P^2 \# P^2$ of two projective planes can be represented as a square disk with sides identified according to the word $aabb$.
   (b) Prove that the $P^2 \# P^2$ is homeomorphic to a Klein Bottle.
3. Identify the resulting surfaces (up to homeomorphism) if the edges of a square are identified according to the following words.
   (a) $aa^{-1}bb^{-1}$
   (b) $abab$
   (c) $a^{-1}bab$
4. Let X be a topological space, and $A \subset X$ have the relative (or "subspace") topology. Prove the following theorems:

   Theorem A set $C \subset A$ is closed in A if and only if it is the intersection of A with a closed set of X.

   Theorem If $B \subset A$ and $\overline{B}$ denotes the closure of B in X, then the closure of B in A equals $\overline{B} \cap A$.
5. Consider $\mathbb{Q}$ as a subset of $\mathbb{R}$, where the real numbers are given the standard topology. Prove that the closure of $\mathbb{Q}$ equals all of $\mathbb{R}$.
6. Let $X = \mathbb{N}$. Let $\mathcal{S} = \{X_i \,|\, i = 1, 2, 3, \ldots\}$ denote the collection of subsets X_i, where $X_i = \mathbb{N} - \{i\}$. Prove that $\mathcal{S}$ is a subbasis on X. Describe the resulting basis and topology on X.
7. Let $\mathcal{B} = \{[a, b) \,|\, a < b,\, a, b \in \mathbb{R}\}$, where $[a, b) = \{x \,|\, a \le x < b\}$ as usual. Prove that $\mathcal{B}$ is a basis for a topology on $\mathbb{R}$. Is this topology finer or coarser than the standard topology on $\mathbb{R}$? (In other words, does it have more or fewer open sets than the standard topology?)
8. Prove the statement, or find a counter-example to show it is false.
   (a) $\overline{A} \cap \overline{B} = \overline{A \cap B}$
   (b) $\overline{A} \cup \overline{B} = \overline{A \cup B}$

Note that some of the above problems were assigned on homework, and you've been given solutions. Others were examples in class and/or your book.