This writeup should give you an idea of what I would look for when grading this problem. It is is *not* intended to be a perfect solution. In particular, you could sketch a better picture by hand in about two minutes.

#14. Evaluate the given integral by changin to polar coordinates: $\int\int_D x\ dA$. where D is the region in teh first quadrant that lies between the circles $x^2+y^2=4$ and $x^2+y^2=2x$.

The graph of $x^2+y^2=4$ is a circle of radius 2, centered at $(0,0)$. In polar coordinates, this equation becomes simply $r=0$.

The second equation requires a bit more work. To convert it to a polar equation, we substitue $r\cos\theta$ for x and $r\sin\theta$ for y:

$$\begin{aligned} r^2\cos^2\theta + r^2\sin^2\theta &= 2r\cos\theta \\ r^2 &= 2r\cos\theta \\ r &= 2\cos\theta \end{aligned}$$

This is the polar equation for a circle of radius 1, centered at the point $(x,y)=(1,0)$. (This can be seen directly from the original equation by completing the square, which results in the equation $(x-1)^2+y^2=1$.)

The portions of these circles which lie in quadrant one are shown in Figure 1. To describe the region between the circles in polar coordinates, we can let θ range from 0 to $\pi/2$. Our values for r should range from the smaller circle to the larger one, as depicted by the dotted

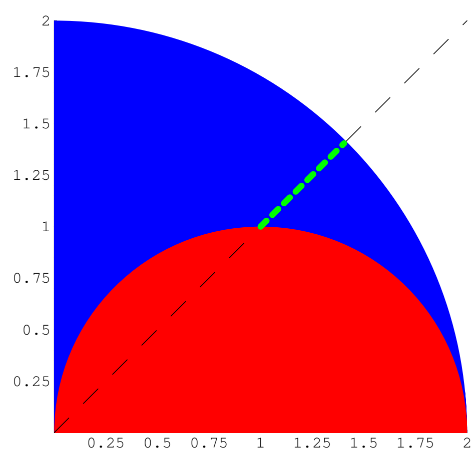

FIGURE 1. (a) The portions of the two circles in the first quadrant

line in the figure.

Recalling again that $x = r\cos\theta$ and $dA = r dr d\theta$, our integral becomes

$$\begin{aligned}
\int\int_D x\,dA &= \int_0^{\pi/2}\int_{2\cos\theta}^{2} r^2\cos\theta\,drd\theta \\
&= \int_0^{\pi/2}\left[\frac{r^3}{3}\cos\theta\right]_{r=2\cos\theta}^{r=2} d\theta \\
&= \frac{8}{3}\int_0^{\pi/2}\cos\theta - \cos^4\theta\,d\theta \\
&= \frac{8}{3}\left([\sin\theta]_0^{\pi/2} - \int_0^{\pi/2}\cos^4\theta\,d\theta\right) \\
&= \frac{8}{3}\left(1 - \left[\frac{1}{32}(12\theta + 8\sin 2\theta + \sin 4\theta)\right]_0^{\pi/2}\right) \qquad \text{(table of integrals)} \\
&= \frac{8}{3}\left(1 - \frac{3\pi}{16}\right) \\
&= \frac{8}{3} - \frac{\pi}{2}
\end{aligned}$$

We can get a rough check of this answer as follows. The area of the region D is easily computed to be $\frac{\pi}{2}$. The average value of the function x in the region is somewhat less than 1, since D is thicker on the left. Doing these sorts of visual estimates can be very tricky, but let's say the average value of x in D is about 0.70. Thus

$$\int\int_D x\,dA \cong \frac{\pi}{2}\cdot 0.70 \cong 1.09956$$

Since $\frac{8}{3} - \frac{\pi}{2} \cong 1.09587$, this estimate agrees quite well with our answer.