

1. (a) Consider the surface given by  $x^3y - yz^2 + z^5 = 9$ . Use a gradient to find the plane tangent to this surface at the point  $(3, -1, 2)$ .  
 (b) Find the points on the surface  $x^4 + y^4 + z^4 = 1$  where the tangent plane is parallel to the plane  $x + y + z = 1$ .  
 (c) Find the points at which the ellipsoid  $x^2/4 + y^2 + z^2 = 1$  is tangent to one of the hyperboloids in the family  $x^2 + y^2 - (z + 1)^2 = c^2$ .
2. In the region given by  $0 \leq x \leq \pi$ ,  $0 \leq y \leq \pi$ , find all the points on the curve  $\sin(x) + \cos(y) = 0$  parallel to the line  $y = x$ .
3. The gradient of a function is given by  $(x, y^3, z)$ .  
 (a) Sketch the intersection of its level sets with the plane  $z = 1$ .  
 (b) Describe where the level sets are tangent to the plane  $x + 4y + z = 0$ .
4. (a) Find an expression in terms of  $n$  for the points on the curves  $x^{2n} + y^{2n} = 1$ , which have tangents parallel to the line  $y = -2x$ .  
 (b) Sketch this sequence of points in the  $xy$ -plane indicating any limits of such sequences of points.
5. (a) Find all the points on the unit sphere,  $\{(x, y, z) : x^2 + y^2 + z^2 = 1\}$  whose tangent planes contain the vector  $\mathbf{j} + \mathbf{k}$ .  
 (b) Find the linear transformation from the unit sphere to the surface  $S$  given by  $x^2 + (y^2)/4 + z^2 = 9$ .  
 (c) Use your answers to parts a and b to find the set of points on the surface  $S$  whose tangent planes contain the vector  $(2\mathbf{j} + \mathbf{k})$ . Be careful to explain why your method gives you the correct answer. Checking points in the set is not sufficient for full credit.
6. Any and all of Barr, p150 #6-10.
7. (Barr, p150 #21) Show that if  $a$ ,  $b$ , and  $c$  are all nonzero and  $A = \begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$  then  
 $p(\mathbf{x}) = \mathbf{x}^T A \mathbf{x}$  is indefinite.
8. Any and all of Barr, p238 #15-18.
9. Let  $f(x, y) = \cos x \cos y$ . Find the second order Taylor polynomial for  $f$  at  $(a, b) = (0, 0)$ .
10. To which entry in the matrix is the value of the determinant  $A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$  most sensitive?

11. Find all critical points of the following functions, and classify the behavior at those points.

(a)  $f(x, y) = x^2 + xy + y^2 + 2x - 2y + 5$

(b)  $g(x, y, z) = x^3 + xy^2 + x^2 + y^2 + 3z^2$

(c)  $h(x, y) = 4 - 3x^2y^2$

(d)  $i(x, y) = x^2y^3z^4$

(e)  $j(x, y) = 2 - x^4y^4 - z^4$

12. What point on the plane  $3x - 4y - z = 24$  is closest to the origin?

13. Find the extrema of  $f(x, y) = x^2/4 + y^2$  subject to the condition that  $x^2 + y^2 = 1$ .