Math 2374

Overview : Chapter 1 and 2 (except 1.4, 2.2)

This review sheet is not meant to be your only form of studying. Understanding all the homework problems and lecture material are essential for success in the course. This review sheet only contains the key ideas of these sections.

Chapter 1:

1. Computing 2×2 and 3×3 determinants. Knowing their geometric meanings.
2. Dot products and cross products and their corresponding geometric meanings.
3. Vectors in $\mathbb{R}^n$. Be able to find magnitudes of vectors.
4. Matrices. Multiply matrices times vectors, matrices times matrices.
5. **Parametric equation** of a line through point (x_0, y_0, z_0) in direction of $\langle a, b, c \rangle$:

$$\begin{cases} x = x_0 + ta \\ y = y_0 + tb \\ z = z_0 + tc \end{cases}$$

6. **Equations of planes:** The equation of plane contains a point (x_0, y_0, z_0) and has a normal vector $\langle a, b, c \rangle$:

$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

or

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

7. Distance from a point (x_0, y_0, z_0) to a plane $ax + by + cz + d = 0$:

$$\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Chapter 2.

2.1: The geometry of real-valued functions.

1. Knowing graph of a function, Level sets, and Sections.
2. Level sets: We call level curves for functions of two variables; and level surfaces for functions of three variables. Be able to sketch a few level sets as in the homework.

2.3 : Differentiation.

1. Partial derivatives:
   (a) Understand and compute partial derivatives.
   (b) Methods: limit definition, one-variable calculus techniques.
2. The derivatives:
   (a) The derivatives of a function is represented by the matrix of partial derivatives.

(b) **Linear approximation** of a function $f : \mathbb{R}^2 \to \mathbb{R}$ near a point (x_0, y_0).

$$z = f(x_0, y_0) + \left[\frac{\partial f}{\partial x}(x_0, y_0)\right](x - x_0) + \left[\frac{\partial f}{\partial y}(x_0, y_0)\right](y - y_0).$$

(c) A function is differentiable at point x_0 means it is nearly linear around that point.

(d) Suppose all partial derivatives exist and are continuous near a point x_0, then f is differentiable at x_0.

(e) For a scalar-valued function f, the derivative $Df(x)$ can be written as vector $\nabla f(x)$, the **gradient**.

(f) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable at (x_0, y_0). **Tangent plane** of the graphs of f at point $(x_0, y_0, f(x_0, y_0))$ is

$$z = f(x_0, y_0) + \left[\frac{\partial f}{\partial x}(x_0, y_0)\right](x - x_0) + \left[\frac{\partial f}{\partial y}(x_0, y_0)\right](y - y_0).$$

2.4: Paths and curves.

1. Know that a curve can be parametrized by a function $c(t)$. Also, $c'(t)$ is the velocity of an object at position $c(t)$. Moreover, $c'(t)$ is **tangent vector** to the curve at time t.
2. Be able to compute the tangent line to a curve.

2.5: Properties of the derivative.

1. (Product Rule)
$$D(fg)(x) = g(x)Df(x) + f(x)Dg(x)$$
2. (Quotient Rule)
$$D\left(\frac{f}{g}\right) = \frac{g(x)Df(x) - f(x)Dg(x)}{[g(x)]^2}$$
3. (Chain Rule)
$$D(f \circ g)(x) = Df(g(x))Dg(x)$$
4. Suppose $c(t)$ is a path and a function $f : \mathbb{R}^3 \to \mathbb{R}$. Then
$$D(f \circ c)(t) = Df(c(t))Dc(t) = \nabla f(c(t)) \cdot c'(t).$$

2.6: Gradients and Directional derivatives.

1. The gradient ∇f:

   (a) For scalar-valued function f, the gradient ∇f is like the matrix of partial derivatives Df, except that the gradient is a vector rather than a matrix.

   (b) The gradient is a vector whose magnitude and direction have physical meaning:
   - The gradient points in the direction where f increase most rapidly.
   - the magnitude of the gradient indicates the rate of change in f in that direction.

   (c) For $f : \mathbb{R}^n \to \mathbb{R}$. The gradient ∇f is perpendicular to level sets of f, we can use the gradient to find tangent planes to surfaces.
   For example, when $n = 3$, the tangent plane of the level surface $f(x, y, z) = k$ (k is a constant) at point (x_0, y_0, z_0) is
   $$\nabla f(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0.$$

2. The directional derivative:

   (a) The directional derivative $D_v f$ gives the rate of change of f in direction v.

   (b) $D_v f(x) = \nabla f(x) \cdot v$ where v is a **unit vector**.